“Interest rate trap”, or: Why does the central bank keep the policy rate too low for too long time? ⋆

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Abstract

This paper provides a framework for modeling the risk-taking channel of monetary policy, the mechanism how financial intermediaries' incentives for liquidity transformation are affected by the central bank's reaction to financial crisis. Anticipating central bank's reaction to liquidity stress gives banks incentives to invest in excessive liquidity transformation, triggering an “interest rate trap” — the economy will remain stuck in a long lasting period of sub-optimal, low interest rate equilibrium. We demonstrate that interest rate policy as financial stabilizer is dynamically inconsistent, and the constraint efficient outcome can be implemented by imposing ex ante liquidity requirements.

JEL classification: E5, G21, G28

Key words: Interest rate trap, Risk-taking channel, Systemic risk, Liquidity requirements, Macroprudential regulation

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1 Introduction

Since 2007, central banks worldwide have responded to the global financial crisis with massive, persistent cuts in interest rates. Whereas the ECB recently raised rates slightly, the Fed announced in August 2011 that it "*anticipates that economic conditions are likely to warrant exceptionally low levels for the federal funds rate at least*" for two years. At the same time, quite a few observers are concerned about the long run impact of these measures. Prominent critiques warned from early on against a policy of low interest rates and called for an early "end of the era of ultra-low rates" (Rajan, 2010, see also Caruana, 2011). They fear that low rates may induce excessive risk-taking, thus increasing the probability of future crises.

According to this view, recent monetary policy measures (such as extensive lender of last resort activity and the commitment to an extended period of keeping interest rates low) risk to create severe distortions. Periods of low interest rates are seen to induce high risk activities in the financial sector, aggravating imbalances in the economy. This argument has particular force since prominent proponents of this view have been lone voices warning against the build up of imbalances after the crash of the new economy (Rajan, 2005 and the Borio & White from the BIS, 2003). Their concern is motivated by the experience of that period, as emphasized by Rajan (2010): "*After all, the low rates introduced by Alan Greenspan from 2002-2004 created momentum in house prices that soon became the rationale for crazy lending. Put differently, by the time risk-taking and asset price inflation again take off, it may be too late for the Fed to turn it back.*"

Right after the crash of the new economy, Illing (2004) argued already in a similar vain that the Fed might be forced to keep interest rates low for an extended period — out of fear that in view of highly leveraged intermediaries, higher rates might pose a risk for the stability of the financial system. Low interest rates, in particular in combination with an explicit, transparent announcement to keep them low in the future, may increase incentives for financial intermediaries to invest in fragile activities, tying the hands of the central bank. Since raising rates in a severely weakened financial system might trigger another financial meltdown, the actions taken in the financial sector will force central banks to keep interest rates low again. Taken hostage by the financial industry, central banks thus may end up in what we call an "Interest Rate Trap" — the economy remains stuck in a long lasting period of sub-optimal, low interest rate equilibrium.

Our paper tries to address this phenomenon and to analyze adequate policy designs. According to the "risk-taking channel" of monetary policy transmission, liquidity and risk-taking are tightly interconnected. In this paper, we focus on the incentives to invest in excessive liquidity transformation. For that reason, we model pure liquidity risk (the model can easily be extended to allow also for solvency risk, see Cao, 2010). Extending Cao & Illing (2011), we consider an economy with a fragile financial system with banks providing liquidity transformation services via nominal deposit contracts. Banks as financial intermediaries offer deposit contracts which can be withdrawn at any time with sequential service constraint. This mechanism works as disciplinary mechanism to commit banks not to abuse their superior collection skills. So in our economy, financial fragility is motivated from first principles, via incentive constraints such as in Diamond & Rajan (2005). In order to allow for the possibility of an interest rate trap (a situation where the central bank is forced to keep interest rates low for an extended period), we analyze a repeated setting of Cao & Illing (2011), simplified to the case that systemic shocks are extremely rare events.

In an infinite time horizon, there is a continuum of investors born at each period t, all with an initial endowment of one unit of resources. Investors live up to two periods. When they are young (at t), they can deposit their funds at bank accounts. The banks, which are infinitely lived, use these funds to finance projects run by entrepreneurs. One generation entrepreneurs are born at each period t, and live up to 3 periods. Entrepreneurs have specific skills, required to carry out the project i with return $R_i > 1$ successfully. Type 1 project are liquid; they yield a return R_1 the following period; type 2 projects yield a higher return $R_2 > R_1$ but they are possibly illiquid — their realization may be delayed one more period. Replacing an entrepreneur before returns are paid out would yield only a share $\gamma < 1$ of the project's return. Thus, due to their bargaining power, entrepreneurs cannot commit to payout more than the share γ to investors. They keep the remaining share as rent. Banks as financial intermediaries can pool funds across different projects and have superior collection skills compared to investors (a higher γ). Deposit contracts with a sequential service constraint work as a commitment mechanism for banks not to abuse their superior skills, forcing them to pay out all the funds collected from entrepreneurs. Otherwise, they would be run by investors. Bertrand competition among banks drives bank's profits to zero in equilibrium.

All agents are assumed to be risk neutral. Investors are "impatient"; they urgently need to consume the return of their funds when they get old (at $t + 1$). Then at $t + 1$, a new generation of investors are born with initial endowment which they again can deposit at bank accounts. In contrast, entrepreneurs are "patient"; they are indifferent between consumption in the second or third period of their lives. The assumption of all investors being impatient is the most straightforward way to model a need for liquid assets as a limit case of a more general setting with only some random share of investors being impatient.

At each period t banks choose how much funds to invest both in liquid type 1 projects (yielding a safe return $R_1 > 1$ in the following period), and in type 2 projects (with higher return $R_2 > R_1 > 1$). Type 2 projects are possibly illiquid; they may be realized with a delay — only after two periods. On the aggregate, a share of type 2 projects will be realized early, the others being available only with delay. The share of liquid type 2 projects is random, not known at the time of investment. It can take on two values: With probability π, the share will be high (p); otherwise it will be low ($\underline{p}$) with $\underline{p} < p$. We assume that the state — call it crisis state — with a low share ($\underline{p}$) realized early is an extremely low probability event, that is probability $\pi \rightarrow 1$.

Banks compete à la Bertrand among investors by choosing some investment strategy (characterized by the share α invested in type 1 projects). Investors can observe all banks investment strategies and will deposit their funds at the bank offering the highest expected return for deposit contracts. To maximize investor's payoff, banks need to cut down on investment in liquid, low return projects. They invest only so much in those projects that they are able to payout depositors for sure as long as no systemic liquidity event occurs. This way, they can take advantage of the high return of type 2 projects and cut down on investment in liquid projects. So they pick the share α^* in type 1 projects which maximizes investor's return in the case that aggregate risk does not materialize, rationally discounting the possibility of a systemic shock with insufficient aggregate liquidity as a rare, low probability event. Patient entrepreneurs with successful projects realized early are willing to provide liquidity by depositing their rents at banks at the equilibrium gross interest rate, 1. In equilibrium, all banks follow the same strategy (see Cao & Illing, 2011).

If, however, $\underline{p}$ is realized with a large share of type 2 projects being delayed, there will be a liquidity shortage: Banks will not be able to payout all depositors as promised, having no longer sufficient liquid funds. Liquidity dries out in that case, driving up the real rate of interest such that the banks will not be able to survive. Since information about that systemic event is already publicly available at an intermediate stage before implementing the projects, banks will be immediately run by investors and are forced to liquidate all projects (of both type 1 and type 2), just paying out an inferior return $c < 1$. Since this systemic event has low probability (π being close to 1), strategy α^* maximizes investors return, given the probability π and given the constraint that deposit contracts act as incentive mechanism, creating financial fragility (making bank prone to the risk of runs).

But bank runs being extremely costly, in case of a systemic shock there are strong arguments for public intervention to prevent inefficient liquidation of all projects, triggered by a systemic run. This is exactly what central banks are doing, acting as lender of last resort. With nominal deposit contracts, they can inject additional (paper) liquidity, supporting the struggling banks. Following Allen & Gale (1998) and Allen, Carletti, & Gale (2011), we model lender of last resort policy in the following way: In case of a systemic shock, the central bank provides additional paper money to the bank. Lender of last resort policy prevents interest rates from shooting up in case of systemic risk, ensuring that banks are always able to pay out their nominal commitments. By keeping the interest rate low, there is no longer an incentive for investors to run the bank, eliminating inefficient liquidation. The central bank's liquidity injection drives up the aggregate price level such that the real value of nominal deposit contracts is equal to the real resources available in the economy. At first sight, this policy helps to raise investor's payoff compared to the no intervention equilibrium, implementing the constrained efficient outcome in our economy.

Unfortunately, however, as we show in section 3, this policy on its own can have detrimental long run impact. Once the central bank intervenes in a systemic crisis, the economy will get stuck in a low interest rate trap, with inferior outcome for investors for the following reason: Banks anticipating the central bank's reaction have an incentive to cut down completely on private provision of liquidity. When investing for the next period, they will choose $\alpha = 0$ rather than $\alpha = \alpha^*$, investing all their funds in illiquid projects. They realize that this way, the central bank will be forced to keep its interest rate low, independent of whether there is another systemic shock or not in the future. Relying on the central bank as lender of last resort, private banks deliberately create excessive financial fragility, forcing the central bank to continue its generous support. In the unique subgame perfect equilibrium, all banks will no longer invest in private provision of liquidity, driving down investor's expected return to an inefficiently low level and forcing the central bank to continue its low interest rate policy.

A policy of announcing a path of increasing interest rates in order to scare financial intermediaries to reduce their exposure to financial fragility will not be credible, since it is not dynamically consistent: Banks will not believe such an announcement, rationally anticipating central bank's incentive come to rescue again in order to avoid a costly systemic bank run. As long as the interest rate is the only instrument for the central bank to ensure financial stability, time inconsistency problem cannot be solved. There is no way to deter excessive risk taking by raising the policy rate, since rational financial intermediaries deliberately keep the economy in a highly fragile state.

Nevertheless, as we show, in our economy there is a simple way to escape the unfortunate interest rate trap: Imposing adequate liquidity requirements ex ante is sufficient to prevent the economy being stuck in a highly fragile state. Assume that a regulator allows banks to be active only in case they invest sufficient funds in private provision of liquidity (that is, imposing the liquidity regulation $\alpha \geq \alpha^*$). Then, as long as the economy stays in the normal state (with p being realized), there will no longer be a liquidity shortage. Guided by prudential regulation, banks have a strong incentive to invest enough funds in liquid assets in order to pay out all depositors in normal times. In case the rare systemic event occurs, the central bank can safely act as lender of last resort, avoiding costly bank runs.

Related literature

Triggered by the financial crisis, a new active research field analyzes the link between monetary policy, financial stability concerns and the perception and pricing of risk by economic agents. Borio & Zhu (2008) argue that changes in the financial system and in regulation had a profound impact on the relation between central bank policy and risk taking incentives of financial intermediaries, changing the way monetary policy affects the real side of the economy compared to traditional transmission mechanisms. They point out that standard macroeconomic models do not take proper account of these changes. Borio & Zhu suggest that there is a need to establish an additional new channel, which they call the "risk-taking channel" of monetary policy. They identify several ways how a policy of low interest rates might contribute to building up imbalances, singling out (1) leverage (working via balance sheet effects like the financial accelerator as modeled in Adrian & Shin, 2011), (2) search for yield (related to distorted management incentives) and (3) transparency (the way central bank communicate policies and their reaction function), in particular perceived commitment to future policy decisions. In that context, they emphasize the perception that the central bank reaction function is effective in cutting off large downside risks, by "censoring" the distribution of future outcomes.

Borio & Zhu (2008) argue that liquidity and risk-taking are tightly interconnected, and can reinforce each other. Our model focuses exactly on the incentives for liquidity transformation. We explicitly take into account the central bank's reaction to financial crisis: lender of last resort policy acts as insurance against downside risks, thus introducing an asymmetry in bank's payoffs. We show that anticipating central banks reaction results in excessive liquidity transformation. We analyze a rational expectation (dynamically consistent) equilibrium, so there is full transparency about the future (contingent) interest rate path. But since there is a unique dynamically consistent equilibrium, central banks cannot affect the outcome by being less transparent about future policy. Financial intermediaries, rationally anticipating central bank's reaction, will not behave differently when the central bank would announce a (not credible) less accommodating policy. It turns out that the straightforward way to cope with distortionary incentives is strict ex ante liquidity regulation rather than interest rate policy.

Most of the literature on the "risk-taking channel" is either descriptive (such as Rajan, 2005 and Borio & Zhu, 2008) or empirical. Several econometric studies have found evidence of a risk-taking channel. Jimenez, Ongena, Peydró & Saurina (2011) use a micro-level data set for Spanish banks. They analyze the impact of short term interest rates set by Deutsche Bundesbank on credit risk-taking of Spanish banks during the period before the introduction of the Euro. They show that with low rates, banks act more aggressively: When rates are low, they lend to borrowers with a worse credit history and grant more loans with a higher per-period probability of default. They point out that the moment of highest credit risk is when short-term rates that were very low for a long time period suddenly increase steeply: At that moment, the net worth of an already risky pool of borrowers is suddenly eroded possibly leading to non-performance and defaults. They argue that reducing interest rates again lowers the credit risk of all outstanding loans, making banks more willing to accept even more credit risk thereby reducing the tensions in the credit markets. Similar results have been found in a study of bank risk-taking in Bolivia during the period of a peg to the US dollar with a highly dollarized financial system (Ioannidou, Ongena & Peydró, 2009).

The closest work related to ours are the papers by Giavazzi & Giovannini (2010) and Diamond & Rajan (2011). Giavazzi & Giovannini (2010) build on work of Tirole (2011) to model the interactions between monetary policy and liquidity transformation. They claim that “optimal” monetary policy consists of a modified Taylor rule which takes into account the possibility of liquidity crises and so adjusts to the risk taking channel. According to Giavazzi & Giovannini (2010), failure to recognize this point risks leading the economy into a low interest rate trap: Low interest rates induce too much risk taking and increase the probability of crises. These crises, in turn, require low interest rates to maintain the financial system alive. Raising rates becomes extremely difficult in a severely weakened financial system, so monetary authorities remain stuck in a low interest rates trap.

Giavazzi & Giovannini (2010) presents various interesting examples illustrating the problems with market fragility associated with liquidity transformation. In our view, however, the examples presented in their paper do not properly address the key issue — the nature of and the policy response to a low interest rate trap. Just like Giavazzi & Giovannini (2010), we focus on financial fragility arising from incentives for excessive liquidity transformation due to central bank insurance as lender of last resort. We model the role of financial fragility from first principles. In contrast to their claim, however, we find that strict ex ante liquidity regulation is the optimal policy design rather than a (non-credible) attempt to raise interest rates above the natural rate.

Diamond & Rajan (2011) present a theoretical model close to our setting with a central bank (or, in their model rather a fiscal authority) being able to affect real interest rates. Arguing that ex ante regulation may not be effective if the extent of ex ante promises are hard to observe, they suggest that raising real interest rates in normal times may prevent banks to make excessive liquidity promises despite the propensity of the social planner to reduce rates in crisis times. In their model, liquidity demand arises from a positive future supply shock, raising the "natural" real rate. Since agents prefer to smooth consumption, they would to like to raise consumption already early, creating a demand for liquidity with liquidation of long term projects. Via non-distortionary taxation and redistribution of household endowments, the central bank is able to improve upon the market outcome. Diamond & Rajan (2011) do not address the feasibility of taxation. In our model, giving the social planner taxation power cannot help to overcome the underlying frictions. The reason is that additional resources from taxation are simply not available at the time when liquidity constrained investors urgently need them. As we show, the prospective public liquidity provision via taxation will destroy the market mechanism of private liquidity supply. In our view, the adequate policy response it to address the underlying causes of the distortions (incentives for excessive liquidity holding) rather than following a third best fiscal policy, which in general runs the risk to introduce additional distortions with non-lump-sum taxation.

Structure of the paper

The paper is orgainzed as follows. In section 2, we start from the baseline model with real financial contracts, then characterize the social planner's constrained efficient solution as well as the market equilibrium outcome. In section 3 we show that central bank as the lender of last resort can eliminate inefficient bank failures by lowering its policy rate, but such policy is subject to the time inconsistency problem. The unique dynamic consistent equilibrium is that the banks engage in excessive liquidity risk and the central bank's policy rate is kept too low for too long time. In section 4, we argue that giving the central bank some fiscal authority, for instance, the power to tax the economy, may not be the right way to tackle the problem, due to the nature of liquidity crisis. However, ex ante liquidity regulation, combined with the central bank's commitment to providing liquidity at low interest rate, can fix the dynamic inconsistency problem and implement the first-best allocation. Section 5 concludes.

2 The Baseline Model

In this section, we consider a non-monetary model without central bank. Therefore, all financial contracts are written in real terms.

2.1 The agents, time preference, and technology

Consider the following economy populated by three types of risk neutral agents: investors and entrepreneurs in overlapping generations, as well as infinitely lived banks. The economy extends over infinite time horizon, $T = \{0, 1, \ldots, t, \ldots\}$ (the details of timing will be explained later). It is assumed that:

(1) There are a continuum of investors born in each period $t \in T$, call them generation t investors. Each investor lives up to 2 periods — "young" and "old": She is endowed with one unit of resources when she is young, the resource can be either stored (with a gross return equal to 1) or invested in the form of bank deposits; she consumes when she is old. There is no population growth;

(2) There are a finite number N of active banks engaged in Bertrand competition for investors' deposits. The banks as financial intermediaries can fund projects of entrepreneurs using these deposits;

(3) There are a continuum of entrepreneurs born in each period $t \in T$, call them generation t entrepreneurs. Each entrepreneur lives up to 3 periods — "young", "middle age", and "old": She works on one project (call it project t) when she is young, then consumes the proceeds later; however, she is indifferent in the timing of consumption. There are two types of entrepreneurs (denoted by i, $i = 1, 2$), distinguished by the returns of their projects R_i:
   (a) Type 1 projects (safe projects) are realized one period after investment, with a safe return $R_1 > 1$;
   (b) Type 2 projects (risky projects) yield a higher return $R_2 > R_1 > 1$. They are risky in the sense that the returns can be delayed: With probability p_t,

these projects will be realized one period after investment, but they may be delayed (with probability $1 - p_t$) until two periods after investment. The exact value of p_t, however, is not known at the time of investment, t. It will only get revealed between periods t and $t + 1$ at some intermediate date, call it $t+0.5$. In the aggregate, the share p_t of type 2 projects initiated at t will be realized early. In the rest of this paper, we are more interested in the case of aggregate shocks. We model them in a simple way: The aggregate share of type 2 projects realized early, p_t, can take on just two values — either p or $\underline{p}$ with $\underline{p} < p$. The "normal" state with a high share of early type 2 projects p, i.e., the state with plenty of liquidity, will be realized with probability $\pi \to 1$, and the "crisis" state with a low share of early type 2 projects $\underline{p}$. This captures the fact that the crisis state with aggregate liquidity shortage is a rare event. In the following, we assume that $1 < \underline{p}R_2 < pR_2 < R_1$ to focus on the relevant case. We will explain this later.

To focus on the case where liquidity constraints are binding, we assume that there are more projects of each type available in each period than the aggregate endowment of investors so that resources of investors are scarce. Therefore, in a frictionless market economy without commitment problems (to be explained in the next paragraph), total surplus would go to the investors: They simply put all their funds in early projects and capture the full return. However, the commitment problems prevent realization of the frictionless market outcome, and this creates a demand for liquidity. Since there is a market demand for liquidity only if investors' funds are the limiting factor, we choose the investors' payoff as the policymaker's objective and concentrate on deviations from this market outcome.

The commitment problems come from hold-up problems as modeled in Hart & Moore (1994), or Holmsträm & Tirole (1997): Entrepreneurs can only commit to pay a fraction $\gamma < 1$ of their return (assume that $\gamma R_1 > 1$). The role of banks as financial intermediaries is justified by their superior collection skills to the investors' (or, the banks have a higher γ). In the following, we also assume that $p < \gamma$ to concentrate on the relevant case that investors care about investment in liquid projects (see section 2.3). Following Diamond & Rajan (2001), banks offer generation t investors deposit contracts at period t with a fixed payment d_{0t} payable at any time after t as a credible commitment device not to abuse their collection skills. The threat of a bank run disciplines bank managers to fully pay out all available resources pledged in the form of bank deposits. Deposit contracts, however, introduce a fragile structure into the economy: Whenever investors have doubts about their bank's liquidity (the ability to pay investors the promised amount d_{0t} at $t+1$), they run on the bank at the intermediate date, forcing the bank to liquidate all its projects (even those funding entrepreneurs with safe projects) at high costs: Early liquidation of projects gives only the inferior return $c < 1$. In the rest of this paper, we do not consider pure sunspot bank runs of the Diamond & Dybvig (1983) type. Instead, we concentrate on the runs happening if liquid funds are not sufficient to pay out investors.

2.2 *Timing and events*

As Figure 1, at t the newly born investors receive their endowments. The banks compete for the investors' deposits by offering deposit contracts that promise a fixed payment d_{0t} which maximizes the expected return of investors. Banks compete by choosing the share α of deposits invested in type 1 projects, taking their rivalries' choice as given, and the share $1-\alpha$ in type 2 projects. The value of α is public information so that investors have rational expectations about each bank's return from the projects. Since the banks' problem is symmetric in each period, we drop the time subscript for α whenever it doesn't lead to confusion.

Figure 1 Appears Here

At $t+0.5$, the value of p_t is revealed, so is the expected return of the banks at $t+1$. A bank will experience a run if it cannot meet the investors' demand. In this case, all the projects — even the safe ones — have to be liquidated. At $t+1$ those banks which do not suffer a run can roll over their debts by trading with early entrepreneurs in a perfectly competitive market for liquidity: Since entrepreneurs retain a share $1-\gamma$ of the projects' returns, the banks can borrow from the entrepreneurs (precisely, type 1 entrepreneurs and a share p_t of type 2 entrepreneurs whose projects return early) at $t+1$, using the long assets (those risky projects that return late) as collateral. The liquidity market is cleared by the equilibrium borrowing rate r_{t+1}. Banks use the liquidity provided and the collected proceeds from the projects to pay out investors. In this way, impatient investors can profit indirectly from the investment in high-yielding long-term projects. So banking allows the transformation between liquid claims and illiquid projects.

At $t + 2$, the banks collect the return from the late t projects and pay back the generation t early entrepreneurs at the predetermined interest rate r_{t+1}.

The same procedure is repeated from $t+1$. After generation t investors' withdrawing deposits and consuming, generation $t + 1$ investors are born with endowments. Again, the banks compete for the investors' deposits by offering fixed payment deposit contracts, and use the funds to finance both safe and risky projects of generation $t + 1$ entrepreneurs. Then at $t + 1.5$ the share of risky projects that return early, p_{t+1}, is revealed (assume that p_t at $t+0.5$ and p_{t+1} at $t+1.5$ are i.i.d.). If a bank does not experience a bank run, it borrows from generation $t+1$ early entrepreneurs at $t + 2$ to maximize the payment to generation $t + 1$ investors. Again, at $t + 3$ the banks collect the return from the late $t + 1$ projects and repay generation $t + 2$ early entrepreneurs. And so on. The timing structure is summarized in Figure 2.

In addition, when a bank experiences a bank run and gets dissolved at $t + 0.5$ ($\forall t \in T$), we assume that the bank can be restructured into a new bank at $t + 1$ and restart its business. Since the signals at intermediate dates, such as p_t and p_{t+1}, are i.i.d., such new virgin bank doesn't bear any competitive disadvantage comparing with the other banks.

FIGURE 2 APPEARS HERE

2.3 The constrained efficient allocation

The reference point, or the constrained efficient allocation of the model, is taken from the solution to a social planner's problem maximizing the investors' return. Assume that the planner has the same collection skill (the same γ) as the banks. Given that the investors have a short time preference on consumption, for generation t investors, the planner will have to choose the optimal level of α that maximizes aggregate liquidity supply at $t+1$, which is captured by the following proposition:

Proposition 2.1 *The optimal solution for the planner's problem is*

(1) *At any t the planner invests the share $\alpha = \frac{\gamma - p}{\gamma - p + (1-\gamma)\frac{R_1}{R_2}}$, on safe projects, and generation t investors' expected return at $t+1$ is $\gamma\left[\alpha R_1 + (1-\alpha)R_2\right]$;*

(2) *When $\underline{p}$ is realized at $t + 0.5$, generation t investors' return at $t+1$ is $\alpha R_1 + \underline{p}(1-\alpha)R_2 < \gamma\left[\alpha R_1 + (1-\alpha)R_2\right]$.*

Proof Since the central planner's problem is symmetric at any t, we can simply drop the time subscript in the following. At any t the central planner maximizes the investors' return by setting α for each bank such that

$$\alpha = \arg\max_{\alpha\in[0,1]} \gamma\left\{\alpha R_1 + (1-\alpha)\left[pR_2 + \frac{(1-p)R_2)}{r}\right]\right\},$$

with $r \geq 1$. Solve to get $\alpha = \frac{\gamma - p}{\gamma - p + (1-\gamma)\frac{R_1}{R_2}}$, with $r = 1$. □

2.4 The market equilibrium

Since market equilibria are symmetric at any t, we can simply drop the time subscript in the following. The market equilibrium is characterized by the representative bank i's strategy chosen at any $t \in T$, (α_i, d_{0i}), $\forall i \in \{1, \ldots, N\}$, such that

(1) Taken the other banks' strategic profiles as given, bank i maximizes its profit from its investment made at t:

$$\alpha_i = \arg\max_{\alpha_i\in[0,1]} \gamma\left\{\alpha_i R_1 + (1-\alpha_i)\left[pR_2 + \frac{(1-p)R_2)}{r}\right]\right\};$$

(2) Bank i makes zero profit from offering generation t investors deposit contract d_{0i}, or d_{0i} is equal to the bank's expected return from investing the investors' deposits:

$$d_{0i} = \max_{\alpha_i\in[0,1]} \gamma\left\{\alpha_i R_1 + (1-\alpha_i)\left[pR_2 + \frac{(1-p)R_2)}{r}\right]\right\};$$

(3) The liquidity market at $t+1$ is cleared by the equilibrium interest rate r, which is determined by the aggregate liquidity supply and demand:

$$r = \max\left\{\frac{\sum_{i=1}^{N}\gamma(1-\alpha_i)(1-p)R_2}{\sum_{i=1}^{N}(1-\gamma)\left[\alpha_i R_1 + (1-\alpha_i)pR_2\right]}, 1\right\}.$$

Aggregate liquidity demand and supply being jointly determined by all the banks' investment on liquid assets, or α_i, the optimal strategy for the banks is to choose the proper α_i that minimizes their refinancing cost, or r. The market equilibrium is featured by the following proposition:

Proposition 2.2 *The constrained efficient allocation can be decentralized by the market equilibrium under the realization of p_t, with $r = 1$ for $t+1$, $\forall t \in T$.*

(1) In such equilibrium, all the banks set $\alpha^* = \frac{\gamma - p}{\gamma - p + (1-\gamma)\frac{R_1}{R_2}}$ and generation t investors' expected return at $t+1$ is $d_0^* = \gamma[\alpha^* R_1 + (1-\alpha^*)R_2]$;

(2) If $\underline{p}$ is realized at $t+0.5$, the banks suffer from bank run and have to immediately liquidate all their assets. Investors' return at $t+0.5$ is c, with $c < 1 < \alpha^* R_1 + \underline{p}(1-\alpha^*)R_2 < d_0^*$.

Proof It is easy to verify claim (1), using similar argument as Cao & Illing (2011). Further, it is not difficult to see that the investors run on the banks at $t+0.5$ once the crisis state gets revealed, since the maximal funds the banks can raise at $t+1$ is not sufficient to meet the deposit contract, $\alpha^* R_1 + \underline{p}(1-\alpha^*)R_2 < d_0^*$.

When the crisis state gets revealed at $t+0.5$, the banks can raise funds from (1) borrowing from generation t early entrepreneurs at $t+1$ with interest rate $\tilde{r} \geq 1$, plus (2) postpone a fraction $d_0^* - \left[\alpha^* R_1 + \underline{p}(1-\alpha^*)R_2\right]$ of the repayments to generation $t-1$ early entrepreneurs to $t+2$. However, since generation $t-1$ entrepreneurs will die after $t+1$, (2) will not work. The maximum funds the banks can raise is therefore $\gamma\left[\alpha^* R_1 + (1-\alpha^*)\underline{p}R_2 + \frac{(1-\alpha^*)(1-\underline{p})R_2}{\tilde{r}}\right] \leq \alpha^* R_1 + (1-\alpha^*)\underline{p}R_2 < d_0^*$. Anticipating that the banks won't be able to meet the deposit contract at $t+1$, generation t investors will run on the bank at $t+0.5$ so that all the assets need to be liquidated. The investors only obtain the liquidated value c. □

Note that by assuming $\pi \to 1$ we abstract from the banks' free-riding incentive discussed in Cao & Illing (2011) so that we can focus on the risk-taking channel induced by the central bank policy. Now the only inefficiency in the market equilibrium, comparing with the solution to planner's problem, comes from the costly bank runs under $\underline{p}$. In the next section, we discuss the effect of central bank's lender of last resort policy which attempts to eliminate the bank failure and improve the efficiency.

3 Lender of Last Resort Policy and the Interest Rate Trap

In this section, we consider the role of central bank in preventing the costly bank failure. The central bank cannot directly mobilize real resources, but it can ease the banks' liquidity constraints by providing liquidity at a favorable interest rate. The liquidity injection from the central bank takes the form of fiat money, and it follows the Bagehot principle that the banks can borrow at the policy rate r_t^M ($\forall t \in T$) against their assets as collateral. With the introduction of central bank, the economy of the model becomes a monetary one. From now on, we assume that the setup of the model remains the same as the baseline case except that all the financial contracts are nominal.

Since the market equilibrium is consistent with the constrained efficient solution as long as the economy is in the normal state, there is no role for the central bank. Therefore, at any t that follows a normal state signal $p_{t-0.5} = p$ the central bank should set r_t^M above some threshold value, call it $\bar{r}$ (will be determined later), to induce the constrained efficient allocation. Similarly, if the normal state is revealed at $t + 0.5$, the central bank should stay with $r_{t+1}^M|p > \bar{r}$ to induce the constrained efficient allocation when the banks are making their investment decisions at $t + 1$. However, if the crisis state $\underline{p}$ is revealed at $t+0.5$, the central bank's optimal policy is to lower the interest rate, i.e., set $r_{t+1}^M|\underline{p} = 1$ to allow the banks borrow the fiat money at the lowest cost. This eliminates the costly bank run, as the following proposition shows. Now suppose that at any t with the normal state the banks believe that the central bank will stay with $r_{t+i}^M > \bar{r}$ ($i = 1, 2, \ldots$) as long as the state of the world is normal, so that they all set α^* at t. If this is the case, Proposition 3.1 says that the central bank intervention in the crisis state will avoid the costly bank run and achieve the constrained efficiency.

Proposition 3.1 *Suppose that at any t the banks believe that the central bank can commit to* $r_{t+1}^M|p > \bar{r}$ *and* $r_{t+1}^M|\underline{p} = 1$, *where* $\bar{r} = \frac{(1-p)R_2}{\alpha^*(R_1-R_2)+(1-p)R_2} > 1$. *Then constrained efficient allocation is achieved such that all the banks set* (α^*, d_0^*) *at t.*

When $\underline{p}$ is revealed at $t + 0.5$, the central bank's optimal policy is to lower its interest rate to $r^M_{t+1}|\underline{p} = 1$. This eliminates bank run and restores the constrained efficient allocation.

Proof To repay generation t investors in the normal state, besides the proceeds collected from the early t projects, the banks need to raise funds from (1) trading with early generation t entrepreneurs in the liquidity market, or (2) borrow from the central bank, using the late t projects as collateral. The highest return that the banks can repay the investors through (1) is d^*_0, as Proposition 2.2 shows. The highest return raised through (2) is achieved by maximizing the value of collateral, i.e., when $\alpha = 0$, and such return is $\gamma p R_2 + \frac{\gamma(1-p)R_2}{r^M_{t+1}|p}$. To induce the banks to follow the market solution (1), the policy rate should be so high that $\gamma p R_2 + \frac{\gamma(1-p)R_2}{r^M_{t+1}|p} < d^*_0$. The cutoff rate is therefore determined by $r^M_{t+1}|p > \bar{r} = \frac{(1-p)R_2}{\alpha^*(R_1-R_2)+(1-p)R_2} > 1$.

The investors' return is maximized when the banks get liquidity injection at the lowest cost, so it is optimal for the central bank to set $r^M_{t+1}|\underline{p} = 1$ in the crisis state. Since the banks are facing pure illiquidity problem, they are able to borrow from the central bank up to the collateral value, $\frac{\gamma(1-\alpha^*)(1-p)R_2}{r^M_{t+1}|\underline{p}}$. Therefore, in the crisis state, one bank's nominal worth of total liquidity is $\gamma\left[\alpha^* R_1 + \underline{p}(1-\alpha^*)R_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r^M_{t+1}|\underline{p}}$, which is exactly d^*_0, sufficient to meet the deposit contract with generation t investors. Although the real return for the investors is $\alpha^* R_1 + \underline{p}(1-\alpha^*)R_2 < d^*_0$, it is still better off for them to wait till $t+1$ than to run on the bank at $t+0.5$, knowing that the central bank will lower the policy rate at $t = 1$ and $\alpha^* R_1 + \underline{p}(1-\alpha^*)R_2 > c$. $\square$

Once the central bank commits to stepping in to bail out the banks in the crisis, the situation becomes interesting. Note that after fulfilling generation t investors' claims in nominal terms at $t + 1$, the banks will start the next round investment decisions, conditional on their expectation of the central bank's interest rate policy in the future. Seemingly, the central bank's interest rate policy should depend on the signal at $t + 1.5$ as argued by many, such that the policy rate should rise once the economy starts its recovery from the crisis — in our model this means $r^M_{t+1}|p > \bar{r}$ and $r^M_{t+1}|\underline{p} = 1$. If this were true, the banks should stay with the same optimal (α^*, d^*_0) as stated in Proposition 2.2. And by Proposition 3.1, the corresponding outcome would be constrained efficient.

Unfortunately, this cannot be equilibrium. To see this, suppose that one bank i

sets $\tilde{\alpha} < \alpha^*$ and $\tilde{d}_0 = \gamma[\tilde{\alpha}R_1 + (1 - \tilde{\alpha})R_2]$ at $t + 1$ while the others still choose (α^*, d_0^*). Then the bank will have liquidity problem at $t + 1.5$ even if the normal state gets revealed. In this case, it is ex post optimal for the central bank to stay with $r_{t+2}^M = 1$ — even in the normal state — to avoid the bank's failure. But this means the total liquidity that bank i can raise at $t + 2$ is $\tilde{d}_0 = \gamma[\tilde{\alpha}R_1 + (1 - \tilde{\alpha})R_2] > \gamma[\alpha^* R_1 + (1 - \alpha^*)R_2] = d_0^*$. Therefore, by setting $(\tilde{\alpha}, \tilde{d}_0)$ at $t + 1$, the deviator offers higher nominal return for generation $t + 1$ investors and outbids all its rivalries. Knowing this, all the banks have the incentive to cut down their investment in liquid assets. The only equilibrium is that no bank will invest in any liquid assets and the central bank stay with $r_{t+2}^M = 1$, unconditional on any signal at $t+1.5$. In other words, it is *the* equilibrium that central bank is trapped by the low interest rate, keeping the policy rate too low for too long time, and the banks take excessive liquidity risks!

The formal result is presented in the following proposition:

Proposition 3.2 *After implementing the lender of last resort policy, $r_{t+1}^M|\underline{p} = 1$, the central bank will stay with the low interest rate for the future, i.e., $r_{t+2}^M = 1$, unconditional on the signal observed at $t + 1.5$. All the banks choose $\alpha = 0$ at $t + 1$, and generation $t + 1$ investors are worse off at $t + 2$ than in the market equilibrium.*

Proof See Appendix. □

Proposition 3.2 says that once the central bank commits to stepping in by lowering the policy rate in the crisis state, the banks will use the chance to build up excessive liquidity risk so that any central bank's attempt to raise the policy rate in the future will be seen as a "threat to economic recovery." This is the moral hazard endogenously growing from the central bank's passive bailout guarantee. Such risk-taking channel increases systemic liquidity risk, sowing the seeds of the next crisis: by choosing $\alpha = 0$ at $t + 1$, the central bank is forced to rescue the banks by keeping the policy rate low, no matter whether it is in normal or crisis state!

Furthermore, note that we started from the scenario that the central bank intervenes at $t + 1$ given that the crisis state gets revealed at $t + 0.5$, while we neglected the banks' strategy before $t + 1$. However, the conclusion of Proposition 3.2 can be extended to the period prior to $t + 1$: Since the central bank's commitment to setting policy rate conditional on the state of the world is incredible, the banks will choose $\alpha = 0$ at t even if the current policy rate is high, $r_t^M > \bar{r}$. And the ex post optimal policy for the central bank is to keep the policy rate low after $t + 1$, $r_{t+1}^M = r_{t+2}^M = 1$, unconditional on the state of the world. The ex ante optimal policy rule, such that the central bank starts with $r_t^M > \bar{r}$ and sets r_{t+1}^M and r_{t+2}^M conditional on the states of the world, is not implementable. The equilibrium of the entire game can be summarized in the following corollary:

Corollary 3.3 *With the presence of the central bank's lender-of-last-resort policy — that the central bank commits to lowering its policy rate in the crisis state and raising the rate in the normal state — the unique equilibrium is featured by*

(1) The banks choose $\alpha = 0$ for all $t + i$, $i = 1, 2, \ldots$, even if the initial policy rate $r_t^M > \bar{r}$;
(2) The central bank keeps the policy rate low for the entire future, i.e., $r_{t+1}^M = r_{t+2}^M = \ldots = 1$.

Proof The banks' and central bank's decision problems at t are symmetric as those at $t + 1$. Therefore, the subgame perfect equilibrium profile at t is the same as that at $t + 1$. $\square$

4 Discussion

Can fiscal policy (taxation) work as solution?

Our paper presents the dilemma central banks are facing when exercising their role as lender of last resort in a dynamic context: Anticipating that the central bank will be ready to lower its policy rate whenever there is liquidity stress banks have a strong incentive to engage in excessive liquidity risk irrespective of the initial policy rate. One proposal to solve this problem is to combine monetary and fiscal policy: As suggested by Diamond & Rajan (2011), with the authority to tax the economy, the central bank may be able to improve the allocation in the crisis, achieving a third best solution.

Diamond & Rajan (2011), however, point out that one cannot just focus "on central bank actions, but also (on) where they get real resources from." Modeling the source of taxation carefully in our model, it turns out that such a third-best policy won't work. The reason is straightforward: the prospective public liquidity provision destroys the incentive for private supply of liquidity — an effect that usually has been neglected in the literature on public liquidity intervention. One exception is Bolton et al. (2009). Here, the authors show (in a completely different setup) that public liquidity provision through collateralized lending has the side effect of crowding out private outside liquidity. As we demonstrate in this section, exactly the same problem arises with the taxation solution. To see this, suppose that a signal of crisis state is revealed at $t+0.5$ and the central bank attempts to alleviate the problem via lump-sum taxation of the returns from projects t and $t-1$ at $t+1$. This reduces the effective real return of early generation $t-1$ entrepreneurs to less than 1 (remember that banks need to roll over their debt at t via borrowing from early generation $t-1$ entrepreneurs at the gross interest rate 1). Anticipating this, early generation $t-1$ entrepreneurs will demand to withdraw their deposit at $t+0.5$, exacerbating the banks' liquidity shortage and thus triggering a systemic bank run at $t+0.5$. In other words, the prospective public liquidity provision via taxation destroys completely the market mechanism of private liquidity supply, making the situation even worse!

Liquidity regulation and monetary policy

Our model suggests that interest rate policy suffers from severe time inconsistency problem whenever monetary policy is required for liquidity provision as lender of last resort. The adequate response is to impose liquidity regulation ex ante — at the stage before banks engage in taking excessive liquidity risks. Current reforms in the Basel III framework try to address the insufficient incentives for private liquidity provision. With the new Basel III framework, liquidity requirements will become one of the pillars of the global standard for banking supervision. According to the new rules, in the future banks will be required to meet the liquidity coverage ratio (LCR) — they must hold some share of high quality liquid assets to withstand a certain stressed funding scenario. Up to now, there has been hardly any theoretical analysis to motivate the need for such a ratio. As far as we know, our paper presents the first model providing a framework to analyze the impact of liquidity coverage ratios. In our model, the time inconsistency problem can be solved by imposing a LCR rule ex ante. Supported by adequate liquidity regulation, monetary policy can be implemented in a time consistent optimal way. To see this, suppose now the policy rule is

(1) At any period t, as an entry condition, all the banks are required to hold at minimum share α^* of liquid assets in their investment portfolio. The rule stipulates that at least a share α^* of the deposits have to be invested on the safe projects;
(2) At any period $t + 1$, the central bank commits to setting its policy rate conditional on the observed signal p_t at $t + 0.5$, such that $r^M_{t+1}|(p_t = p) > \bar{r}$ and $r^M_{t+1}|\left(p_t = \underline{p}\right) = 1$.

At any period t, if the normal state is realized, with policy rate being high $r^M_{t+1}|(p_t = p) > \bar{r}$ banks are induced to implement the market equilibrium as stated in Proposition 2.2. The equilibrium is such that all the banks will choose α^*, just meeting the LCR requirement. In contrast, if the crisis state is realized, with policy rate being low $r^M_{t+1}|\left(p_t = \underline{p}\right) = 1$ banks are able to borrow from the central bank and so can meet their nominal contracts with generation $t - 1$ investors. Central bank support eliminates the costly bank run and achieves the constrained efficiency. At the same time, all banks are still required to meet the stipulated LCR when they invest the deposits from generation t investors. This prevents the banks from engaging in excessive liquidity risk, and makes the policy rule dynamic consistent.

In reality, liquidity problems in banking do not only come from the demand side as modeled in our paper - here liquidity crisis arises when banks are not able to fulfill the investors' demand. Liquidity problems may also come from the supply side where the banks' long term projects are financed by short term funding sources (for example, in this paper, the banks' long term risky projects are funded by the investors' short term deposits). In Basel III, a complementary rule — net stable funding ratio (NSFR) — has been introduced to address the supply side issues. NSFR intends to incentivize banks to use stable sources of funding in order to limit the liquidity mismatches, especially to minimize their reliance on short term whole sale funding — a key mechanism that dragged the market stress into a full-fledged crisis after the collapse of Lehman Brothers. Even though in this paper we focus on the demand side problem, our model may be extended by integrating more funding sources to capture the supply side problem in order to shed light on how the NSFR mechanism works. We leave this to future research.

5 Conclusion

Our paper provides a framework for modeling the risk-taking channel of monetary policy. We analyze how financial intermediaries' incentives for liquidity transformation are affected by the central bank's reaction to financial crisis. We model pure liquidity risk in an economy with a fragile financial system with banks providing liquidity transformation services via nominal deposit contracts. Financial fragility may trigger bank runs resulting in inefficient liquidation of resources when a negative systemic shock occurs. In order to prevent inefficient liquidation, the central bank, acting as lender of last resort, injects additional (paper) liquidity.

Since lender of last resort policy acts as insurance against downside risks, it introduces an asymmetry in bank's payoffs. We show that anticipating central banks reaction gives banks incentives to invest in excessive liquidity transformation. This triggers an "Interest Rate Trap" — the economy will remain stuck in a long lasting period of sub-optimal, low interest rate equilibrium. We show that there is a unique dynamically consistent equilibrium. With banks investing in inefficiently low in liquid assets, the central bank is forced to keep the policy rate too low for too long. Anticipating this reinforces the incentives for financial intermediaries to cut down on private provision of liquidity, directly creating systemic risk. Relying on the central bank as lender of last resort, private banks deliberately create excessive financial fragility, forcing the central bank to continue its generous support.

We show that investors are strictly worse off in the dynamically consistent equilibrium. There is no way to deter excessive risk taking by raising the interest rate, since rational financial intermediaries deliberately keep the economy in a highly fragile state. As we show, fiscal policy (taxation) cannot implement the constrained efficient outcome in our model. In contrast, the constrained efficient outcome can be implemented by imposing regulatory rules, such as a liquidity coverage ratio (LCR) ex ante. With the adequate liquidity coverage ratio (LCR) imposed, the central bank can act as lender of last resort in case a systemic shock occurs.

Appendix

A Proof of Proposition 3.2

The proposition is proved by backward induction. The game consists of two subgames:

(1) When the bankers choose $\left(\alpha^*, d_0^*\right)$ at $t+1$,

  (a) If the central bank stays with the low interest rate at $t+2$, the banks' nominal return is d_0^*;

  (b) If the central bank raises interest rate at $t+2$, the banks' nominal return is d_0^*, given that the probability of crisis state at $t+2$ is very low.

(2) When the bankers choose $\left(\tilde{\alpha}, \tilde{d}_0\right) \neq \left(\alpha^*, d_0^*\right)$ at $t+1$,

  (a) If the central bank stays with the low interest rate at $t+2$, the banks' nominal return is $\tilde{d}_0 = \gamma[\tilde{\alpha}R_1 + (1-\tilde{\alpha})R_2]$ which is maximized at $\tilde{\alpha} = 0$. In this case, $\tilde{d}_0 = \gamma R_2 > d_0^*$;

  (b) If the central bank raises interest rate at $t+2$, the banks' nominal return is $\tilde{d}_0 = \max\left\{\tilde{\alpha}R_1 + (1-\tilde{\alpha})pR_2, \gamma\left[\tilde{\alpha}R_1 + (1-\tilde{\alpha})pR_2 + \frac{(1-\tilde{\alpha})(1-p)R_2}{r_{t+2}^M}\right]\right\} < d_0^*$.

The unique subgame perfect equilibrium is that the bankers choose $\alpha = 0$, $d_0 = \gamma R_2$ at $t = 1$, and the central bank stays with $r_{t+2}^M = 1$, unconditional on the signal observed at $t + 1.5$. However, the real return the investors get is $pR_2 < d_0^*$. $\square$

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Figure 1: Timing of the model (investors)

$\boldsymbol{t}$	$\boldsymbol{t+0.5}$	$\boldsymbol{t+1}$	$\boldsymbol{t+1.5}$	$\boldsymbol{t+2}$
Gen. $t-1$: Withdraw and consume				
Gen. t: Deposit in the banks	p_t gets revealed; decide whether to run	If no run, withdraw and consume		
		Gen. $t+1$: Deposit in the banks	p_{t+1} gets revealed; decide whether to run	If no run, withdraw and consume
				Gen. $t+2$: Deposit in the banks

Figure 2: Timing of the model (banks and entrepreneurs)

$\boldsymbol{t}$	$\boldsymbol{t+0.5}$	$\boldsymbol{t+1}$	$\boldsymbol{t+1.5}$	$\boldsymbol{t+2}$
Banks: Collect late returns of proj. $t-2$; repay early gen. $t-2$ ent.; collect early returns of proj. $t-1$; liqudity trade with early gen. $t-1$ ent.; repay gen. $t-1$ investors; contract with gen. t investors; invest $(\alpha, 1-\alpha)$ on proj. t	If experience run, liquidate & exit; otherwise continue	Collect late returns of proj. $t-1$; repay early gen. $t-1$ ent.; collect early returns of proj. t; liquidity trade with early gen. t ent.; repay gen. t investors; contract with gen. $t+1$ investors; invest $(\alpha, 1-\alpha)$ on proj. $t+1$	If experience run, liquidate & exit; otherwise continue	Collect late returns of proj. t; repay early gen. t ent.; collect early returns of proj. $t+1$; liquidity trade with early gen. $t+1$ ent.; repay gen. $t+1$ investors; contract with gen. $t+2$ investors; invest $(\alpha, 1-\alpha)$ on proj. $t+2$
Entrepreneurs: Late proj. $t-2$ mature; early gen. $t-2$ ent. repaid; early proj. $t-1$ mature; gen. $t-1$'s liquidity trade with bank; get loans for proj. t	If experience run, all projects terminated; otherwise continue	Late proj. $t-1$ mature; early gen. $t-1$ ent. repaid; early proj. t mature; gen. t's liquidity trade with bank; loans for proj. $t+1$	If experience run, all projects terminated; otherwise continue	Late proj. t mature; early gen. t ent. repaid; early proj. $t+1$ mature; gen. $t+1$'s liquidity trade with bank; loans for proj. $t+2$