Forward Guidance in a Simple Model with a Zero Lower Bound

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Abstract

In this paper we present a simple framework to model central bank forward guidance in a liquidity trap. We analyze the role of long–run and short–run price stickiness under discretion and commitment in a straightforward and intuitive way. Despite the impact of price rigidity on welfare being non–linear, losses under discretion are lowest with perfectly flexible prices. We show why the zero lower bound may still be binding even long after the shock has gone and characterize conditions when a commitment to hold nominal rates at zero for an extended period is optimal.

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1 Introduction

Forward guidance is seen as a handy tool to steer the real rate of interest and hence real activity when the nominal policy rate is stuck at the zero lower bound (ZLB). However, this transmission channel strongly depends on the central bank’s credible commitment to future activities - a commitment which central banks are often reluctant to make. The aim of this paper is to provide, in the spirit of Barro and Gordon (1983), a simple setup which summarizes the recent advances on a theoretical foundation of forward guidance, which can, for example, be found in Eggertsson and Woodford (2003) and Werning (2012). Using the framework of Benigno (2009), our aim is to analyze forward guidance within a traceable setup. We characterize the role of long and short run price stickiness under discretion and commitment. We show that the impact of price rigidity on welfare may be non-linear, but losses under discretion are always lowest with perfectly flexible prices. Credible forward guidance depends on the feasibility of ”irresponsible” (Krugman, 1998) monetary easing. We show why the zero lower bound may still be binding even long after the shock has gone and characterize conditions when a commitment is optimal to hold nominal rates at zero for an extended period is optimal. Recently, Cochrane (2013) argues that - due to nominal indeterminacy - the New Keynesian framework exhibits multiple equilibria with different price paths, some of them with mild inflation and no output loss during a liquidity trap. We characterize the optimal price path in our model and show that price stickiness eliminates price level indeterminacy under optimal policy.

2 Model

We consider a three-period setup. We call periods 1 and 2 the short run and period 3 the long run, i.e. after period 3 variables do not change. The households’ optimization problem is given by

\[
\max_{\{C_t, N_t\}_{t=1}^3} \mathbb{E}_1 \left[ \sum_{t=1}^3 \left( \frac{1}{1 + \rho_t} \right) \left( \frac{C_t^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} - \frac{N_t^{1+\varphi}}{1 + \varphi} \right) \right]
\]

s.t.

\[
P_1C_1 + B_1^S + B_1^L = W_1N_1 + T_1
\]
\[
P_2C_2 + B_2^S = W_2N_2 + (1 + i_1^S)B_1^S + T_2
\]
\[
P_3C_3 = W_3N_3 + (1 + i_2^S)B_2^S + (1 + i_1^L)B_1^L + T_3
\]
where $\rho_t$ is the stochastic discount rate, $\sigma$ is the elasticity of inter-temporal substitution, $\varphi$ characterizes the elasticity of labor supply, $C_t$ is consumption, $N_t$ are hours worked and $P_t$ is the price level. Households can save via the purchase of short-term (one period) bonds, $B_t^S$, and long-term (two period) bonds, $B_t^L$, which yield interest of $i_t^S$ and $i_t^L$, respectively. $W_t$ is the nominal wage rate and $T_t$ are lump sum net transfers including firms’ profits. It is straightforward to derive the log-linear aggregate-demand curves

$$y_1 - y^* = E_1[y_2 - y^*] - \sigma(i_1^S - [E_1[p_2] - p_1] - \rho_1)$$ (1)

$$y_2 - y^* = E_2[y_3 - y^*] - \sigma(i_2^S - [E_2[p_3] - p_2] - \bar{\rho})$$ (2)

With perfect credit markets, the arbitrage condition $i_t^L = i_t^S + i_t^S$ must hold.

Firms have mass one. We are interested in the interaction between short–run and long–run price stickiness. To keep the model simple, we assume that a share $\alpha_1$ of firms have to fix their prices for at least two periods in period 0. So they cannot adjust in periods 1 and 2. With probability $\lambda$ each of the $\alpha_1$-type firms may even not be able to adjust their prices in period 3. The parameter $\lambda$ allows us to vary the degree of long–run rigidity in period 3 independent of rigidities in the other periods. A share $\alpha_2$ of firms exhibits short–run price stickiness. They set their price always one period in advance (at $t-1$) before observing shocks occurring during period $t$. We assume that $\alpha_2 > 0$. The remaining $1 - (\alpha_1 + \alpha_2)$ firms can adjust their prices freely at any time. We consider a demand shocks occurring only in period 1. So the $\alpha_2$–types, perfectly anticipating the future price path, will set the same price level in period 2 and 3 as the free–adjusters. Firms production technology is given by $Y_t(i) = A_t N_t(i)$, $\forall i \in [0,1]$, and the good market is monopolistic competitive such that $Y_t(i) = (P_t(i)/P_t)^{-\theta} Y_t$ with $\theta$ being the elasticity of substitution between the continuum of goods. Given this pricing scheme aggregate (log-) supply is described by

$$y_1 - y^* = \frac{1}{\kappa_1}(p_1 - p^*)$$ (3)

$$y_2 - y^* = \frac{1}{\kappa_2}(p_2 - p^*)$$ (4)

$$y_3 - y^* = \frac{1}{\kappa_3}(p_3 - p^*)$$ (5)

where $\kappa_1 = \frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}(\frac{1}{\sigma} + \varphi)$, $\kappa_2 = \frac{1-\alpha_1}{\alpha_1}(\frac{1}{\sigma} + \varphi)$ and $\kappa_3 = \frac{1-\alpha_1 \lambda}{\alpha_1 \lambda}(\frac{1}{\sigma} + \varphi)$. Since $\lim_{\alpha_1 \to 0} \kappa_2 = \lim_{\alpha_1 \to 0} \kappa_3 = -\infty$ but $\lim_{\alpha_1 \to 0} \kappa_1 \neq -\infty$ there will be no output gaps in period 2 and 3 if prices are perfectly flexible. In period 1, however, an output gap emerges independently of $\alpha_1$ since also the $\alpha_2$–type firms have their period 1 prices set to $p^*$.

Monetary policy is characterized by announcing a future price path $\{p_2, p_3\}$ to forward
guide expectations. The central bank’s objective is to minimize the quadratic loss function derived from a second order Taylor approximation of the utility function

\[
\mathcal{L}_1 = \frac{1}{2} \times \mathbb{E}_1 \left[ (y_1 - y^*)^2 + \frac{\theta}{\kappa_1} (p_1 - p^*)^2 + \frac{1}{1 + \rho_1} \left( (y_2 - y^*)^2 + \frac{\theta}{\kappa_2} (p_2 - p^*)^2 \right) + \left( \frac{1}{1 + \rho_1} \right) \left( \frac{1}{1 + \bar{\rho}} \right) \left( (y_3 - y^*)^2 + \frac{\theta}{\kappa_3} (p_3 - p^*)^2 \right) \right]
\]

where \( p^* \) is the steady state price level \( \alpha_1 \)-type firms have anchored their prices to and \( y^* \) is the flexible price output. When minimizing welfare losses the central bank is constrained by the New Keynesian IS–curves (Equations (1)–(2)) and Phillips–curves (Equations (3)–(5)).

For the simulation exercise below we choose a standard calibration with \( \beta = 0.99, \sigma = \varphi = 1, \theta = 5 \). We choose \( \alpha_2 \) to be small to allow for high \( \alpha_1 \) when \( \alpha_1 \to 1 - \alpha_2 \); \( \alpha_2 = 0.1 \). Somewhat arbitrarily we choose \( \alpha_1 = 0.25 \), such that in period 1 approximately one third of firms cannot adjust their prices, and \( \lambda = 1 \) as the impact of these parameters on optimal policy is analyzed below.

### 3 The problem of dynamic inconsistency

To provide the simplest framework, we perform the following thought experiment: before period 1 the economy is in its steady state and the central bank has been expected to stabilize prices at \( \mathbb{E}_0[p_t] = p^*, \ t \in \{1, 2, 3\} \). Following Eggertsson (2006) we assume that in period 1 a negative time preference shock, \( \rho_1 \), with \( \rho_1 < 0 < \bar{\rho} = \rho_2 \), hits the economy and drives it to the zero lower bound. There is no persistence in the shock, so the economy will revert back to normal in period 2. But solely by cutting the interest rate down to zero, the central bank cannot prevent a recession in period 1 since this would require a negative nominal rate. It can, however, announce to raise the price levels in the following periods above \( p^* \) in order to lower the current real rate of interest and thus to stimulate current consumption even when the nominal policy rate remains stuck at zero.

\footnote{To be explicit monetary policy implements the desired aggregate price level \( p_t \) via the announcement of \( p_t^* \), the optimal price level the \( \alpha_2 \)- and \( (1 - \alpha_1 - \alpha_2) \)-types will charge. Thereby it takes into consideration that a fraction \( \alpha_1 \) of firms cannot respond to that announcement.}
To perfectly stabilize the economy in the first period the central bank would need to credibly announce a price level of $\bar{p}_2 = p^* + |\rho_1|$ for period 2. Such a policy, however, cannot be the optimal commitment strategy: raising $p_2$ above $p^*$ will cause inefficiencies and thus welfare loss next period. The optimal commitment strategy is to promise to raise $p_2$ only so much that the marginal loss in period 2 (from accepting a price $p_2 > p^*$) will be just equal to the marginal losses in period 1 (from accepting a price $p_1 < p^*$). Credibility is a crucial feature of forward guidance, since if agents have marginal doubt in the central banks willingness to implement the announced path the strategy unravels. This can be seen in Figure 1 where, for the sake of simplicity, we assume that $p_3 = p^*$ and that the nominal rate of interest, $i^S$ and $i^L$, can be set consistently.

In Figure 1, point $C$ with $(p^*_2, y^C_2)$ characterizes the optimal commitment strategy. However, this commitment solution suffers from the well understood dynamic inconsistency problem: the promise to implement the commitment path is not credible. As soon as the shock has gone and time preference reverts back to $\bar{\rho}$ in period 2, the central bank has an incentive to renege on its promises. After all, at that stage, aggregate demand reverts to normal, so there is no longer any reason to stimulate the economy. The promise to raise the price level anyway implies the central bank is nevertheless willing to shift prices and output beyond target levels. Suppose the central bank wants to raise next period’s price level to $p_2^*$ by shifting the AD curve in Figure 1 upwards to $AD^C$. Ex-ante, from period 1’s perspective, only the prices of $\alpha_1$-types have already been fixed. So the ex-ante relevant AS–curve is the line $DA$ described by Equation (4). Thus, long-run price rigidity as captured by $\alpha_1$ allows forward guidance to raise both price level and output in period 2, despite being fully anticipated. If agents trust that promise, the
expected price level next period will be $E[p_2] = p_2 = \alpha_1 p^* + (1 - \alpha_1) p^*_2$, bringing the economy to point $C$. Once period 2 has been reached, however, the central bank faces a new, flatter $AS^{\text{ex-post}}$ curve, since ex post now also prices of the $\alpha_2$-types are fixed and thus price rigidities are stronger. The ex-post $AS$ curve intersects the flexible price output at $\bar{p} = \frac{\alpha_1}{\alpha_1 + \alpha_2} p^* + \frac{\alpha_2}{\alpha_1 + \alpha_2} p^*_2$. The central bank has an incentive to ignore past statements and instead try to dampen prices. Ex-post optimal policy, given sticky prices, chooses point $A$ rather than $C$.

As long as agents have marginal doubt in the central bank’s commitment to stick to its promise, they will anticipate that incentive already in period 1 and will charge a price below $p^*_2$, thereby reducing $p_2$. Neither will private consumers in period 1 trust that the central bank is willing to implement a high rate of inflation. Being afraid that instead the real rate of interest will stay high, they prefer to save rather than to spend in period 1. So the strategy unravels. In the end the unique discretionary equilibrium is to implement $\{p^*; y^*\}$ at point $D$. Therefore, under discretion the expectation channel breaks down and the central bank is not able to credibly promise any excess inflation in period 2 ($p^*_2 = p^*_3 = p^*$ and $y^*_2 = y^*_3 = y^*$). There is no way to attenuate the adverse effects of the severe recession in period 1 and monetary policy, constrained by the ZLB, will remain too tight. Without any commitment to future activities the shock will hit the economy full tilt with a the real rate being too high, $\rho_1 = r^*_1 < r^*_1 = \frac{\sigma_{\phi}}{1 + \sigma_{\phi}} \rho_1 < 0$, and strong deflation $p^*_1 - p^* = \frac{\rho_1}{1 + \rho_1 \phi} \theta_1 < 0$. Only under credible forward guidance as postulated by Assumption 1 in the next section the central bank can guide agents’ expectations.

In our setup, deflation in period 1 gets worse without any intervention the lower the degree of period 1 price stickiness, $\alpha \equiv \alpha_1 + \alpha_2$. This result is also found by Werning (2012). However, since with flexible prices output losses are lower and deviations of the price level from target have a decreasing weight in the welfare function, aggregate welfare losses are lowest with perfect price flexibility as can be seen in Figure 2. This result is in strong contrast to Werning’s Proposition 2. In our discrete time setup we identify, moreover, a non-linearity in the effect of price rigidity on welfare loss. In contrast to Werning, in our model discretionary welfare losses $L^D_1 = \frac{1}{2} \left( \frac{1 + \theta_1}{1 + \sigma_{\phi}} \right)^2 (\sigma \rho_1)^2$ are decreasing in $\alpha$ only for high degrees of price stickiness. This threshold is reached for $\alpha \geq \bar{\alpha} = \frac{1 + \sigma_{\phi}}{2(1 - \frac{\theta}{2}) + \sigma_{\phi}}$. Since $\alpha \in [0, 1]$, $\alpha$ will always be below $\bar{\alpha}$, if $\bar{\alpha} \geq 1$ or equivalently if $2 \sigma \geq \theta$. Generally, aggregate welfare losses will only start to decrease in the degree of price stickiness for a sufficiently high degree of inter-firm competition. However, even in this case welfare losses always exceed the losses under fully flexible prices. The intuition for this non-linearity is straightforward: for high degrees of price stickiness, prices can hardly deviate from $p^*$. Thus, the effect of a lower deflation due to higher $\alpha$ will at some

The real rate of interest is defined as $r_t = i_t - (E_t p_{t+1} - p_t)$ and $r^*_t$ denotes the natural real rate.
point be stronger than the effect of larger output gaps due to higher $\alpha$.

**Figure 2:** Price Rigidities and Welfare Loss

\[
\text{welfare loss} = \alpha \left( 1 + \sigma \phi \right) \left( 1 - \frac{\sigma}{\theta} \right) + \sigma \phi_0 1
\]

**Notes:** All parameters are set at their baseline calibration and $\rho_1 = -0.01$

## 4 Forward Guidance in a liquidity trap

To derive the optimal price path under forward guidance, we introduce the assumption that forward guidance is credible.

**Assumption 1.** The central bank’s announced price path $\{p_2, p_3\}$ is credible in the sense that

\[\mathbb{E}_t[p_{t+1}] = p_{t+1}, \quad t \in [1, 2]\]

The central bank is assumed to be able to guide the aggregate price level perfectly via the announcements. To solve for optimal policy in a liquidity trap we minimize Equation (6) s.t. Equations (1)–(2), (3)–(5), Assumption 1 and $i_1^S = 0$. The solution is given by

\[
0 = 1 + \theta \kappa_1 \left( p_1 - p^* \right) + \frac{1}{1 + \rho_1} \frac{1 + \theta \kappa_2 \left( 1 + \kappa_1 \sigma \right)}{\kappa_1 \kappa_2 \left( 1 + \kappa_2 \sigma \right)} \left( p_2 - p^* \right) + \ldots
\]

\[
\ldots + \frac{1}{1 + \rho_1} \frac{1}{1 + \rho_1 + \bar{\rho}} \frac{1 + \theta \kappa_3 \left( 1 + \kappa_1 \sigma \right)}{\kappa_1 \kappa_3 \left( 1 + \kappa_3 \sigma \right)} \left( p_3 - p^* \right),
\]

\[
p_1 - p^* = \frac{\kappa_1 \left( 1 + \kappa_2 \sigma \right)}{\kappa_2 \left( 1 + \kappa_1 \sigma \right)} \left( p_2 - p^* \right) + \frac{\kappa_1 \sigma}{1 + \kappa_1 \sigma} \rho_1,
\]

\[
i_2^S = \bar{\rho} + \frac{1 + \kappa_3 \sigma}{\kappa_3 \sigma} \left( p_3 - p^* \right) - \frac{1 + \kappa_2 \sigma}{\kappa_2 \sigma} \left( p_2 - p^* \right)
\]

\[
\text{Dropping the expectation operator, the expected price level in periods 2 and 3 is given by } p_2 = \alpha_1 p^* + (1 - \alpha_1) p_2^* \text{ and } p_3 = \alpha_1 \lambda p^* + (1 - \alpha_1 \lambda) p_3^*, \text{ respectively.}
\]
The first equation of (7) requires optimal policy to equalize marginal welfare losses across time. Thereby monetary policy is constrained by the remaining equations. Since the ZLB is binding in period 1, i.e. \( i_1^S = 0 \), there will be positive co-movement between \( p_1 \) and \( p_2 \), as a higher \( p_2 \) increases inflation between these periods and thus lowers the real rate which stimulates demand in period 1. The short-term nominal rate between periods 2 and 3, \( i_2^S \), and via the no-arbitrage condition \( i_1^L = i_1^S + i_2^S \) also the long-term nominal interest rate, are not necessarily zero as shown in the third equation of (7). In period 1, a discount factor shock disturbs the economy, driving the natural rate below zero. With the ZLB restricting the short-term policy rate, a recession is triggered. While under discretion the economy reverts back to steady state in period 2, optimal policy dampens period 1 recession via promising excess inflation in period 2 forcing the real rate of interest in period 2 below its natural level \( r_n^2 = \bar{\rho} \). Optimal policy allocates period welfare losses optimally across time. Optimally, in the long run (in period 3) the price level reverts to \( p^\star \). This, however, requires deflation between period 2 and 3. To offset deflation, the central bank needs to lower the real rate in period 2 below the natural rate by cutting the nominal rate sufficiently. With households having rational expectations, the real rate of interest is determined by the Fisher equation. Thus, under credible price level guidance the nominal rates have to adjust consistently to the announced price path to satisfy the Fisher equation and to implement certain required real rate. This imposes a crucial constraint on credible forward guidance, since the central bank cannot promise to implement arbitrarily high deflation between periods 1 and 3 (or 2 and 3) as this could require negative nominal interest rates.

Let us first assume that the shock in period 1, \( \rho_1 \), is weak enough such that the ZLB is not binding in period 2. No-arbitrage requires \( i_1^L = i_1^S + i_2^S \) and thus the ZLB is also not binding for \( i_1^L \).

**Assumption 2.** The discount factor shock \( \rho_1 \) is small enough such that under optimal policy the ZLB is not binding on \( i_2^S \), i.e.\(^4\)

\[
|\rho_1| \leq \left( 1 + \frac{1}{1 + \rho_1} \frac{1 + \theta \kappa_2}{1 + \theta \kappa_1} \left( \frac{1 + \kappa_1 \sigma}{1 + \kappa_2 \sigma} \right)^2 \right) \bar{\rho}
\]

Under Assumption 2 we can solve (7) for optimal policy analytically (and derive above constraint). As long as the ZLB is not binding in period 2 the optimal price target in period 3 is \( p_3 = p^\star \) for the following reason: as long as optimal policy is able to dampen the recession by period 2 excess inflation only, there is no need to deviate in period 3 from the target \( p^\star \). In that case, raising the price level in period 3 will have no real effect in period 1 or 2 since \( i_2^S \) adjusts endogenously via the third equation in

\(^4\)Whether the ZLB is still binding in period 2 depends on \( \rho_1 \) and on the other model parameters, in particular price stickiness. Their effects are discussed further below.
So price deviations would only induce only additional welfare losses due to price distortions. Therefore, unconstrained optimal forward guidance implements \( p_3 = p^* \). Figure 3 shows the optimal policy paths compared to the discretionary solution given the baseline parameter calibration and \( \rho_1 = -0.01 \).

**Figure 3: Optimal vs Discretion Policy**

Notes: Unconstrained commitment solution for baseline calibration and \( \rho_1 = -0.01 \) to ensure that the ZLB is not binding for \( i_{S2} \).

Under Assumption 2 optimal forward guidance does not require price deviations from \( p^* \) in period 3. As long as the long–run nominal interest rate \( i_{L1} \) is unconstrained the effect of price deviations in \( t = 3 \) can be absorbed by adjusting the nominal long–run rate.

So, optimal second best policy will bring the economy back to steady state in period 3. With \( p_3 = p^* \) but \( p_2 > p^* \) optimal policy triggers deflationary expectations between periods 2 and 3, which needs to be offset by lowering \( i_{S2} \) to ensure that \( r_2 < r_{n2} = \bar{\rho} \). For large enough shocks, however, the central bank can not credibly promise arbitrary high price level in \( t = 2 \), since this would require a negative \( i_{S2} \). Thus, the ZLB becomes binding also in period 2 for large negative \( \rho_1 \) and for low degrees of price stickiness, \( \alpha_1 \). This is shown in Figure 4 for \( \rho_1 = -0.02 \).
Figure 4: Effect of $\alpha_1$ on optimal policy

Notes: All parameters except $\alpha_1$ are kept at their baseline calibration and $\rho_1 = -0.02$ to ensure that for $\alpha_1 = 0$ the ZLB is binding for $i_2^S$.

The lower the degree of price stickiness, the stronger the excess inflation the central bank aims to implement in period 2. The transmission mechanism is straightforward: the lower the degree of price stickiness, i.e. the smaller the fraction of firms that fixed their prices at $p^*$, the lower the weight of price deviations on welfare losses for $t = 2, 3$. Therefore, an announcement of future price deviations gets less costly for monetary policy. But note that to implement high excess inflation the short–run nominal rate will turn negative in period 2. If so, the announced optimal price path $\{p_2, p_3\}$ cannot be credible since agents anticipate that the nominal rate of interest cannot adjust consistently to this announcement.

Moreover, we see that aggregate welfare losses decrease monotonically along the degree of price stickiness. The reason is that in our model welfare losses due to price deviations occur only because some firms have fixed their prices already in period 0 and cannot adjust thereafter. If the fraction of these firms goes to zero, price deviations in period 2 and 3 are costless and thus monetary policy can stabilize period 1 perfectly. In the absence of long run price stickiness (for $\alpha_1 = 0$), the zero lower bound is no binding constraint even in the first period. In that case, the first best outcome can be

\footnote{Note that $\lim_{\alpha_1 \to 0} \frac{\theta}{\kappa_2} = \lim_{\alpha_1 \to 0} \frac{\theta}{\kappa_3} = 0$.}

\footnote{In this calibration we neglect Assumption 2 for expository purposes and solve for optimal policy without imposing the constraint $\epsilon_2^S \geq 0$.}
implemented via raising $p_2$ to $p^* + |\rho|$. Due to nominal indeterminacy, the price level $p_3$ will be indetermined for $\alpha_1 = 0$.

As shown in Figure 4 for a high degree of price–flexibility but also for large enough shocks, the optimal commitment strategy is not implementable as it would require a negative nominal rate in period 2. We now introduce the ZLB as an additional constraint also for period 2. To this end we replace Assumption 2 by Assumption 3.

**Assumption 3.** The discount factor shock $\rho_1$ is large enough and/or the degree of price stickiness is low such that under optimal policy the ZLB will be binding also in period 2, violating assumption 2. In that case $i_2^S = 0$.

Due to arbitrage, this implies that $i_1^L = 0$. The severity of the shock drives nominal rates to zero and thus restricts monetary authorities in implementing the optimal commitment price path. With the feasible amount of deflation between periods 2 and 3 being limited, policy is now restricted to be third best, requiring $p_3 > p^*$ in order to be able to credibly promise excess inflation in period 2. Thus, in the third best solution the monetary authority must accept welfare losses also in period 3 to stabilize the discount factor shock.

Using $i_2^S = 0$ in (7) allows us to solve for constrained optimal policy analytically. Figure 5 shows optimal policy with the ZLB binding in period 2 compared to unconstrained optimal policy and the discretionary solution for $\rho_1 = -0.05$. Under constrained optimal policy forward guidance can provide less stimulation in period 1. The maximum downward jump in the price path from $t = 2$ to $t = 3$ is constrained by the ZLB on $i_2^S$ as the central bank cannot provide enough nominal ease to make any larger drop credible to agents. This can be seen when looking at the unconstrained solution. Here, optimal policy brings the economy back to initial steady state in period 3 while announcing strong excess inflation in period 2. The corresponding drop in the price level is large enough to drive $i_2^S$ far into negative territory. As agents anticipate that this is not possible, the announced price path is thus not credible and the monetary authority can only implement the constrained best solution which induces higher aggregate welfare losses. Thus, third best policy keeps the short–run nominal rate at the ZLB even after the shock has gone. Note that this is no direct consequence of the shock itself but of the intertemporal trade-off between raising $p_2$ to attenuate the recession and the corresponding deflation between period 2 and 3.

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For the natural long–run real rate to turn negative it is required that $|\bar{\rho} + \rho_1| < 0$. 

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7For the natural long–run real rate to turn negative it is required that $|\bar{\rho} + \rho_1| < 0$. 

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**Notes:** Constrained optimal solution for baseline calibration and $\rho_1 = -0.05$ to ensure that the ZLB is binding for $i_S^2$.

Whereas under unconstrained optimal policy the price path is decreasing between $t = 2$ and $t = 3$, this is not necessarily the case for constrained forward guidance. If period 1 and period 2 prices are very rigid ($\alpha_1 \to 1 - \alpha_2$) but period 3 prices are very flexible, constrained optimal policy can mostly affect period 1 price expectations via period 3 announcements. The optimal price path is then increasing between periods 2 and 3. However, the optimality condition to allocate welfare losses over time still determines the optimal price level in period 3 uniquely.

Extending our model to $n$ periods shows that for $n$ large enough the ZLB will at some point cease being a binding constraint as the necessary deflation can be spread across multiple periods. Then, also third best policy will bring the price level back to $p^*$. The effect of price stickiness on constrained optimal policy is similar to before, as shown in Figure 6. Again, the lower the degree of price stickiness in the model, the more excess inflation will be triggered under constrained forward guidance. For $\alpha_1 = 0$ period 1 can be perfectly stabilized, without any welfare losses occurring over time as the welfare weight on price deviations go to zero. The higher $\alpha_1$ the less accommodative policy is and for $\alpha_1 = 1 - \alpha_2$ barely any excess inflation will be announced. But due to a very flat AS-curve even these small deviations will be very costly as they imply strong output deviations.
Figure 6: Effect of $\alpha_1$ on constrained optimal policy

Notes: All parameters except $\alpha_1$ are kept at their baseline calibration and $\rho_1 = -0.05$ to ensure that the ZLB is binding for $i_2^*$. 

Recently, Cochrane (2013) argues that most results usually found in New Keynesian models during a liquidity trap are artifacts of an arbitrary equilibrium choice. To this end he introduces additional equilibria, identified by different steady state inflation rates that persists once the ZLB stops binding. These equilibria feature price paths that deviate arbitrarily from the old equilibrium path. Our model allows to elaborate on the question how the degree of price stickiness in period 3, the "long–run" period, influences constrained optimal policy. To this end, we introduced the parameter $\lambda$ that determines the degree of price stickiness in period 3 only. For $\lambda = 0$ the period 3 price level is perfectly flexible. Figure 7 shows the effect of period 3 price stickiness on optimal forward guidance policy. We see that independent of the presence of a nominal anchor in $t = 3$ constrained optimal policy determines $p_3$ uniquely. The lower $\lambda$ the larger is period 3 price deviation, as the welfare weight on price deviations approaches zero ($\lim_{\lambda \to 0} \frac{\theta}{\rho_3} = 0$). Consequently, monetary policy can announce stronger excess inflation for period 2, given that $p_3$ can deviate more strongly. Even for $\lambda$ close to zero no arbitrarily large price deviations in period 3 do occur as excess stimulation cannot be optimal as well. The price level $p_3$ is indetermined only for $\lambda = 0$. Hence, with price rigidities, i.e. $\alpha_1 > 0$, (constrained) optimal policy eliminates price level indeterminacy and thus does not support arbitrarily equilibrium choice.
Figure 7: Effect of $\lambda$ on constrained optimal policy

Notes: All parameters except $\lambda$ are kept at their baseline calibration and $\rho_1 = -0.05$ to ensure that the ZLB is binding for $i_s^2$.

5 Conclusion

The implications of our model are manifold: first and well understood, forward guidance is only effective if central banks can credibly commit to future actions (Assumption 1).

Second, if forward guidance is successful, i.e. credible, price expectations are fully determined by the announced price path. This introduces an additional constraint to policy making via the Fisher equation. The nominal interest rate must be set consistently with the announced price path. Optimal policy needs to induce deflationary expectations between periods 2 and 3. Therefore, the monetary authority faces a trade off between promising excess inflation from period 1 to period 2 and the deflation required to bring the price level back to target in period 3. So the amount of excess inflation in period 2 may be constrained by the ZLB even after the shock has already faded away. Understanding this mechanism is of key importance for credible forward guidance. If inflation expectations remain anchored in the US or the Euro Area, as they appear to be, the promise to be irresponsible in the future is less effective and the magnitude of credible irresponsibility is constrained by the ZLB.

Third, we have shown that price stickiness eliminates price level indeterminacy under
optimal policy. Thus, the equilibrium choice, once the discount factor shock abated and the ZLB ceases binding, is not arbitrary but well defined in our model.

Fourth, optimal forward guidance policy aims to bring the price level back to the target price level $p^*$ in period 3. Therefore, and unlike argued by Cochrane (2013), this equilibrium choice is not arbitrary but optimal in our model.

Finally, for high levels of stickiness welfare under discretion may increase with the degree of price stickiness. But in strong contrast to Werning (2012), in our model welfare losses under discretion are always lowest when prices are perfectly flexible.

It is straightforward to extend the model to allow for fiscal policy (the path of government spending or taxation) as commitment device and for more persistent time preference shocks. With the ZLB being binding, the market real rate of interest is above the natural (shadow) rate in period 1. So it will be optimal to shift the path of fiscal policy relative to the optimal first best path by raising government spending (lowering taxes) in the first relative to the second period.

So far we have kept the nominal anchor $p^*$ fixed over time. An interesting extension would be to allow $p^*$ to grow over time, given a certain inflation target $\pi^*$. This would allow us to elaborate on the argument brought forward by Blanchard, Dell’Ariccia, and Mauro (2010) and Ball (2013). They argue that a higher inflation target may increase welfare by reducing the probability of hitting the ZLB in the future. Within our setup, if the $\alpha_1$–type firms adjust their prices ex ante with $\pi^*$, this result follows directly. Given $r^n = \bar{\rho}$ the higher $\pi^*$ the higher will be the steady state nominal rates while at the same time a higher $\pi^*$ is costless in terms of welfare as the nominal anchor growth along with $\pi^*$. If, however, such an adjustment is not feasible, raising the inflation target imposes a trade–off which will be analyzed in future research.

References


