Cyclicality of Wages and Union Power

Annaïg Morin*
Bocconi University, Milan
GATE, University of Lyon II Lumière, Lyon

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Abstract
This paper proposes a dynamic model of the labor market which integrates two main features: matching frictions and trade unions. To examine how trade unions shape the volatility of wages over the business cycle, I decompose the volatility of wages into two components: the volatility of the match surplus and, what constitutes the novelty of the paper, the volatility of the effective bargaining power. Formally, I define the effective bargaining power of the unions as the share of the total surplus allocated to the workers. Starting from the unions’ objective function, I prove that their effective bargaining power is endogenous and countercyclical. Intuitively, because trade unions internalize the impact of a wage increase on the probability for unemployed workers to find a job, they face a trade-off between the wage rate and the employment rate. Therefore, the unions’ preferences (wage-oriented or employment-oriented) fluctuate along the cycle and so does their effective bargaining power. As a result, when the economy is hit by a productivity shock, the dynamics of the unions’ effective bargaining power partially counteracts the dynamics of the total surplus and this mechanism delivers wage rigidity. Relatedly, employment in the union sector reacts more strongly than in the non unionized sector, but its pattern features less persistence.

Keywords: Search and matching; Trade unions; Cycles

JEL classification: J64; J51; E32

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1 Introduction

The role of unions is to protect the rights and interests of their members through representation within firms. It includes the negotiation with employers on behalf of the workers for better wages and working conditions. Therefore, through their direct participation in the wage determination process, unions impact the wage rate level and shapes its volatility. This paper integrates search and matching and trade unions to propose a dynamic framework that accommodates two important sources of wage volatility: the volatility of the match surplus and the volatility of the bargaining power. The paper then studies how these two sources interact over the business cycle to shed new light on the union wage volatility and therefore on the union wage premium volatility.

Specifically, the paper develops a tractable dynamic model that distinguishes between two sectors: a non-unionized sector where wages are negotiated through a traditional bilateral Nash bargaining and a unionized sector where trade unions interact with firms in a collective bargaining process over wages. Formally, I define the effective bargaining power of the workers (in the non-unionized sector) or of the unions (in the unionized sector) as the share of the total surplus allocated to the workers. While the workers' effective bargaining power is fixed in the non-unionized sector, I show that the unions' effective bargaining power is instead endogenous. This result follows directly from the maximization of the unions' objective function. Intuitively, unions being more concerned about the unemployment issue as the labor market becomes slacker, the unions' bargaining power decreases with the unemployment rate. This leads to a double source of wage volatility in the unionized sector: volatility of the match total surplus and volatility of the effective bargaining power. By studying these components of the union wage rate, I make four contributions to our understanding of wage and unemployment fluctuations. First, I propose a framework where the orientation of the union (wage oriented or employment oriented) is endogenous and depends on the relative size of the pool of employed workers within the union. The second result is the countercyclicality of the unions' effective bargaining power. On impact, a positive productivity shock modifies the composition of the union members in favor of the ones fighting for a higher job finding probability (therefore for a lower wage) which leads to a decrease in the bargaining power. As the employment rate increases, the trade union becomes more wage oriented, which explains the steady increase of the bargaining power in the following periods. Third, as a consequence of the second point, the wage rate features some rigidity due to the fact that total surplus and unions' bargaining power fluctuate in opposite directions. I find therefore that a positive productivity shock leads, on impact, to a compression of the union wage premium. This brings us to the fourth point: the moderate reaction of the union wage when the economy is hit by a positive productivity shock translates into a stronger response of employment in this sector. Since the union wage increases with employment, the response of employment in the union sector features less persistence.
This paper builds on Mortensen and Pissarides (1994) search and matching model by introducing trade unions. In doing so, I am able to compare individual wage bargaining and collective wage bargaining. A first study integrating a trade union, in the form of a monopoly union, in a search and matching framework is the one of Pissarides (1986), extended subsequently by Delacroix (2006) and Garibaldi and Violante (2004). These authors model wage determination in the presence of unions and describe the negative impact of unionism on employment due to the union wage premium.

Search frictions make the match valuable by both the firm and the worker, because both parties would be worse off if the match ends, due to the delays of filling a vacancy and finding a new job. This gives employed workers some bargaining power that is used during the wage negotiation to set the wage above the reservation wage, i.e. above the value of unemployment. Noticing that an unemployed worker would accept any wage right above the reservation wage (if he would meet with the firm), one can draw a parallel between this framework and the outsider-insider model. Search frictions can be thought as turnover costs: they induce a delay of filling vacancies, creating a turnover cost, and give the workers the power to set the wage above the reservation wage. In the bisectional model economy developed in this paper, a non unionized sector where wages are negotiated at an individualized level coexists with a unionized sector where unions interact with firms in a collective bargaining process over wages.

The crucial difference between these two sectors can be therefore explained by analyzing the behavior of insiders and outsiders. In an individual wage bargaining, an employed worker, insider, makes full use of his bargaining power to set his wage at the highest possible level considering his bargaining power, without taking into account the negative impact of this high wage rate on the firm’s hiring decision and therefore on the unemployment rate. In contrast, the concern of the trade unions is the welfare of their members, who can be either employed or unemployed. In a collective wage bargaining, the unions internalize this negative impact which harms the outsiders who face more difficulty finding a job when the wage rate is high. The unions do not make full use of their bargaining power because they take into account the fact that the welfare of the insiders (which depends on the wage rate) is negatively correlated with the welfare of the outsiders (which depends on the job finding rate).

The aforementioned papers, incorporating trade unions into search and matching models, propose models in steady state, meaning that they do not assert any change in the wage premium. The model I develop accounts for a dynamic structure of the role of trade union. In doing so, I am able to analyze jointly the volatility of the union wage premium as well as the role of trade union in shaping the volatility of labor market variables. As a start, I demonstrate that the trade union faces a trade-off due to the contradictory demands of employed and unemployed workers, trade-off which is inexistent in a bilateral wage bargaining. Therefore, the orientation of the trade unions (wage-oriented or employment oriented) fluctuates depending on the proportion of each group.
In good times, when the labor market is tight, the proportion of unemployed workers is low, the employment issue is played down, and trade unions push for high wages. In bad times, when the labor market is slack, the trade union moderates the wages in order to boost hirings. This mechanism, that I called composition effect, has an important implication: the union bargaining power is endogenous and is decreasing with unemployment. This finding is consistent with the empirical study of Aronsson, Löfgren, and Wikström (1993) who test two models of wage determination, the first where the bargaining power of the union is constant over time, the second where the bargaining power develops with unemployment and labor market characteristics, using time series data for the Swedish construction sector. They find evidence that unemployment tends to decrease the bargaining power of the union. Two other papers (Campbell III (1997) and Fuess (2001)) confirm empirically this result. Using U.S data, Campbell III (1997) finds that union wages are more sensitive to the unemployment rate than non union wages. Fuess (2001) obtains from Japanese data that the union power is positively correlated with GDP.

Central to my analysis is thus the additional source of wage volatility in the union sector arising from the volatility of the bargaining power. On the one hand, when a positive productivity shock hits the economy, the total surplus from the match goes up and pulls the wage rate up. On the other hand, the composition of the union is modified in favor of unemployed workers who seek for a higher employment probability, which leads to an immediate decrease in the unions’ bargaining power. This last effect translates into a moderate response of the wage rate in the union sector. Moreover, following the productivity shock, employment goes up, leading to a modification of the union’s composition in favor of employed workers pushing for higher wages and to an increase of the bargaining power. The response of the wage rate to the shock is therefore more persistent in the presence of union. As a consequence, employment react more strongly on impact and features less persistence. This model provides therefore a convenient framework to analyze the role of unions in shaping the volatility of wage and employment. In this sense, the present paper is closely related to three papers: Mattesini and Rossi (2007), Zanetti (2007b) and Faia and Rossi (2009). These papers analyze how the response to productivity shocks of employment and wage is affected by the presence of the union and find that the union dampens wage dynamics and amplifies employment dynamics. However, none of these studies integrates trade unions into a search and matching framework, which is yet crucial in order to understand the specific union bargaining power and its volatility, as well as the union wage volatility. Moreover, a search and matching framework provides a natural way of modeling the union’s objective function, from where the endogenous unions’ effective bargaining power is derived, and allows a clear comparison between individual and collective wage bargaining. Finally, the second source of wage volatility, which is at the core of the origin of wage rigidity when wages are collectively bargained, is absent of their analysis.

The paper proceeds as follows. The next section lays out the dynamic equa-
tions of the model. I introduce within a search and matching framework trade unions negotiating wages with firms on behalf of their members and I compare the wage formation process with a standard Nash bargaining setting. Section 3 focuses on how the unions’ effective bargaining power evolves along the cycle and on how this volatility impacts the volatility of union wages. Section 4 analyses the dynamic behavior of the model in case of disturbances (productivity shocks). Section 5 concludes.

2 The model

2.1 The environment

2.1.1 A dual labor market

Labor markets are generally characterized by both individual and collective wage formation. The present model reproduces this feature. Indeed, in the bisectoral economy I model, two producing sectors coexist: the non unionized sector denoted by the superscript $N$ and the unionized sector denoted by the superscript $U$. The unique difference between these two sectors stands in the wage bargaining process. In the non unionized sector, wages are negotiated at an individualized level, i.e. employers and employees agree on wages through a bilateral bargaining process. In the opposite, they are collectively bargained in the unionized sector, meaning that unions negotiate wages with firms on behalf of their members. Indeed, based on the observation that the bargaining process over wages takes mainly place at the sectoral level (OECD (1994)), I make the assumption that all the firms belonging to the unionized sector are engaged in collective bargaining over wages.

Notice that the criteria used to discriminate between the two sectors is the level at which wages are bargained over, not the fact that workers are or not member of a union. In the non unionized sector, wages are individually bargained and workers do not belong to any union. In the unionized sector, wages are collectively bargained but workers are free to decide whether to be unionized or not. The specificity of this sector is that wage agreements obtained by unions do not discriminate between unionized members and non unionized members and this leads to collectively bargained wages for all the employees in this sector. This setup is in line with some empirical studies (Nickell (1997), Nickell and Layard (1999)) which show that a better measure of the unions’ size is provided by the collective bargaining coverage rate, given the presence of excess coverage in several countries. This rate is sluggish (Cahuc and Zylberberg (2004)) which justifies the assumption that the share of workers belonging to each sector is fixed in the short run.

The workers are unable to switch between sectors because each occupation requires specific skills and an extensive investment in training and qualifica-
This segmentation can be brought together with the dual labor market described by Doeringer and Piore (1971) where a primary market with relatively high wages, high degree of concentration, and powerful unions, coexists with a secondary market with opposite characteristics. As presented in MacDonald and Solow (1985), the existence of some sort of barriers prevents free movement across sectors.

### 2.1.2 Timing

At the beginning of the period, an exogenous shock occurs. Existing matches have a probability \( \lambda^i \) (where \( i = N, U \) denotes respectively the non unionized and unionized sector) to end. Subsequently, employees or unions, depending of the sector, bargain with firms over the wage rate. These wage rates, unique in each sector, will be enforced to the new matches. Based on the negotiated wage rate, firms post vacancies and searching firms and unemployed workers meet. This matching process is time-consuming meaning that only a certain percentage of unemployed workers and firms actually form a match each period.

The number of new matches in sector \( i \) at time \( t \), \( m_{it} \), is determined through a matching function increasing in both the size of the pool of unemployed workers searching for a job \( u_i^t \) and the size of the pool of vacancies \( v_i^t \). At the end of the period, the production takes place with a level of employment \( n_i^t \) and salaries are paid. Notice that in this framework, newly employed workers start to produce within the same period. This allows firms to react immediately when facing a shock, in posting more or less vacancies.

Notice that in the present model, the wage negotiation takes place prior to the firms’ hiring decision. This timing contrasts with the one adopted in the search and matching literature, where wages are generally negotiated once the new matches have been hired, but is common practice both in the trade unions literature and in papers integrating trade unions into search and matching frameworks (see Pissarides (1986), Mortensen and Pissarides (1999) and Delacroix (2006)). The right-to-manage approach is based on a sequence à la Stackelberg where employees or unions first bargain with firms over wages and then firms respond by determining employment. If the timing assumption is crucial in the unionized sector, I show in Appendix A that it has no incidence in the non unionized sector, in the special case of a production function with constant return to scale. For reason of comparability, I adopt the same timing.

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1 The utilities sector makes a good example of the unionized sector, whereas the hotel/restaurant/catering sector can represent the non unionized one.

2 See also Silvestre (1971) who uses such a labor market segmentation to describe the French industry since 1945.

3 As presented by Krause and Lubik (2007), decreasing returns (see Cahuc and Wasmer (2001)) and/or downward-sloping demand (see Delacroix (2006) and Ebell and Haefke (2003)) create an intra-firm bargaining effect which makes the timing relevant. Indeed, in each of these two cases, the wage prevailing in a firm is function of the scale of the firm (either through the price level or through the marginal productivity of the worker). Therefore, if the firms decide their employment level before wages are bargained over, they have the possibility to...
in both sectors but it is important to keep in mind that the alternative timing, where wages are negotiated after the matching process, leads to the same equilibrium equations in the non unionized sector.

The labor force is homogeneous and normalized to one, out of which $\alpha^N \in (0, 1)$ belong to the non unionized sector and $\alpha^U = 1 - \alpha^N$ belong to the unionized sector. Employment in sector $i = (N, U)$ at the end of period $t$, $n_i^t$, evolves according to the following dynamics:

$$n_i^t = \alpha^i - u_i^t + m_i^t$$  \hspace{1cm} (1)

whereas the number of unemployed workers at the beginning of period $t$ evolves as:

$$u_i^t = \alpha^i - (1 - \lambda^i)n_{i-1}^t$$  \hspace{1cm} (2)

Plugging (2) into (1), I get:

$$n_i^t = (1 - \lambda^i)n_{i-1}^t + m_i^t$$  \hspace{1cm} (3)

Notice that the law of motion of end-of-period unemployment, $\bar{u}_i^t$, is:

$$\bar{u}_i^t = \alpha^i - (1 - \lambda^i)n_{i-1}^t - m_i^t$$

and that, by the normalization of the labor force:

$$\bar{u}_i^N + \bar{u}_i^U + n_i^N + n_i^U = 1$$

The matching process in sector $i$ is summarized by the following function:

$$m_i^t = \sigma_m u_i^t \sigma_u v_i^t (1 - \sigma_u)$$  \hspace{1cm} (4)

where $\sigma_m$ reflects the matching process efficiency.

The vacancy filling rate is $q_i^t(\theta_i^t) = \frac{m_i^t}{v_i^t}$, where $\theta_i^t = \frac{u_i^t}{v_i^t}$ represents the market tightness, and the job finding rate is $s_i^t(\theta_i^t) = \frac{m_i^t}{u_i^t}$. The matching process presents a congestion externality in the sense that the slacker the labor market, the higher the probability for a vacancy to be filled and the lower the probability for an unemployed worker to find a job.

**2.2 Solving the model**

I solve the model using backward induction. I first determine the optimal response of the firms in terms of hiring decision, taking the wage as given, and then continue backwards to analyze the wage bargaining process.

manipulate the wage through their employment policy. Hence, the intra-firm bargaining effect arises only when the hiring decision is taken prior to the wage negotiation.
2.2.1 Hiring decision

Firms are assumed to be large enough so that by the law of large number the fraction of vacancies filled in each firm is equal to the vacancy filling rate of the sector.

Firms are similar which allows me to focus on a representative firm in each sector. The output in sector $i$ is given by:

$$y^i_t = x_t n^i_t$$

where $x_t$ is the productivity level common to both sectors. The production costs consist of the wage and the cost of posting vacancies ($\kappa$ per vacancy). Given that firms take their hiring decision in a second stage, once the wage rate has been negotiated, the wage schedule is taken as given.\(^4\) The number of posted vacancies results from the following profit’s maximization process:

$$\max_{n^i_t,v^i_t} \pi^i_0 = E_0 \sum_{t=0}^{\infty} \beta^t [x_t n^i_t - w^i_t n^i_t - \kappa v^i_t]$$

s.t. $n^i_t = (1 - \lambda^i)n^i_{t-1} + q^i_t v^i_t$

The first order conditions with respect to $n^i_t$ and $v^i_t$ are given by:

$$x_t - w^i_t - \mu_t^i + E_t \beta \mu_{t+1}^i (1 - \lambda^i) = 0 \quad (5)$$

$$\mu_t^i = \frac{\kappa}{q_t^i} \quad (6)$$

where the Lagrangian multiplier $\mu_t^i$ represents the marginal (and, due to constant return to scale, average) value of employment.

Rearranging the first order conditions I obtain the job creation equation (JC):

$$\frac{\kappa}{q_t^i} = x_t - w^i_t + E_t \beta (1 - \lambda^i) \frac{\kappa}{q_{t+1}^i} \quad (7)$$

This result establishes that firms post vacancies up to the point where the cost of posting a vacancy $\kappa$ times the expected duration of the vacancy $\frac{1}{q_t^i}$ equals the contribution of the worker to the flow of profit plus the vacancy posting cost saved by the firm in $t+1$ in case the match does not end.

2.2.2 Wage negotiation

In a search and matching setting, the allocation of the surplus from the match between the worker and the firm determines the wage bill. The total surplus $S^i_t$ is the sum of the firm’s surplus $S^{f,i}_t = J^i_t$ and the worker’s surplus $S^{w,i}_t = W^i_t - U^i_t$, where $J^i_t$ is the value of employment for a firm, $W^i_t$ is the value of employment for a worker and $U^i_t$ is the value of unemployment. All three are end-of-period values. Notice that, because of constant returns to scale, $J^i_t$ represents the firm’s

\(^4\)This is the reason why the term $\frac{\partial w^i_t}{\partial n^i_t}$ does not appear in the FOCs.
employment value of both the marginal and the average match and $W^i_t$ represents the employment value of both the marginal and the average worker.

These values satisfy:

- Recall from the previous section that the Lagrangian multiplier $\mu^i_t$ represents the value of employment which is denoted here by $J^i_t$. Therefore, it follows from (5) and (6) that:

$$J^i_t = x^i_t - w^i_t + E_t \beta (1 - \lambda^i) J^i_{t+1}$$  \hspace{1cm} (8)

- $W^i_t = w^i_t + E_t \beta [(1 - \lambda^i + \lambda^i s^i_{t+1}) W^i_{t+1} + \lambda^i (1 - s^i_{t+1}) U^i_{t+1}]$ \hspace{1cm} (9)

- $U^i_t = b + E_t \beta [s^i_{t+1} W^i_{t+1} + (1 - s^i_{t+1}) U^i_{t+1}]$ \hspace{1cm} (10)

where $b$ is the unemployment income, potentially including home production.

In both sectors, the wage bargaining process takes place prior to the hiring decision. Therefore, the optimal response of the firms in terms of hiring decision, which is represented by the Job Creation curve, constitutes a constraint for the different participants of the wage bargaining. Said differently, the wage is negotiated by employees or unions and employers who are fully informed of the consequences of their decision on the hiring decision of the firms. I will show in the following paragraphs that this hiring constraint is only binding in the unionized sector.

**Wage determination in the non unionized sector** In this sector an individual bargaining process takes place over the wage rate between employers and employees present at the beginning of the period, before the matching process. The wages being paid after the matching process, the negotiated wage rate applies also to the new matches. Although wages are individually bargained, the equilibrium wage rate is similar for all the workers. The reason for that stands in the homogeneity of the labor force and in the similarity of the firms.

The Nash-bargained wage maximizes the product of the net gains of both parties. At the conclusion of the bargaining, the marginal match provides a welfare $W^N_t$ and $J^N_t$ to, respectively, the worker and the firm, whereas they obtain a welfare of $U^N_t$ and 0 if the bargaining fails. Moreover, the Nash product is maximized subject to the Job Creation curve which represents the optimal response of the firms in terms of hiring decision for any wage rate. Therefore, the Nash bargained wage results from the following maximization program:

$$\max_{w^N_t} [W^N_t - U^N_t] \eta^N [J^N_t]^{1-\eta^N}$$

s.t JC curve: $\frac{z_t}{q_t} = x^i_t - w^i_t + E_t \beta (1 - \lambda^i) \frac{w^i_u}{q^i_{t+1}}$
where \( \eta^N \in [0, 1] \) is the workers’ bargaining power.

It is straightforward from the expressions of \( W^N_t, U^N_t \) and \( J^N_t \) that the Nash product from a marginal bargaining is not function of the level of employment. Therefore, the Job Creation curve, linking the level of employment to the wage rate, is not binding. Intuitively, the workers who are negotiating their wages are not concerned by the hiring process given their status of insiders. They do not take into consideration the impact of the wage rate on the probability for outsiders to find a job. Each of them makes full use of his bargaining power to push his wage at the highest possible level.

The first order condition states that the link between the worker and firm’s surpluses is:

\[
\frac{S^{wN}_t}{S^{fN}_t} = \eta^N \quad \quad \frac{S^{wN}_t}{S^{fN}_t} = \eta^N \tag{11}
\]

Rearranging this first order condition and using the expression of the values \( J^N_t, W^N_t \) and \( U^N_t \), I obtain the equilibrium wage rate in sector \( N \):

\[
w^N_t = \eta^N [x_t + E_t/\beta(1 - \lambda^N)\kappa\theta^N_{t+1}] + (1 - \eta^N)b \tag{12}
\]

The bargaining set is delimited by two threat points and contains an infinity of equilibrium wage rates. The sharing rule allocates a constant share of the bargaining set to the worker and the firm. Unlike in the walrasian model, the wage does not equate the worker’s productivity. Indeed, the wage rate is a function of two labor market parameters: the cost of posting vacancies and the level of unemployment benefits. Moreover, and more importantly, the wage varies with the degree of the labor market tightness in the following period. To see this, consider an increase in the number of posted vacancies in \( t + 1 \). The duration of a vacancy increases, along with the real cost of posting a vacancy \( \kappa \). This translates in period \( t \) into a bigger saving of hiring cost in case of match and therefore into a higher value of employment for the firm and a higher total surplus. The workers surplus being a fixed part of the total surplus, the wage increases. The same reasoning can be followed with an increase of unemployment which leads to a decrease of the worker’s surplus and finally of the wage rate.

The efficiency condition, established by Hosios (1990), entails that the bargaining power of the workers \( \eta^N \) should equal the elasticity of the vacancy filling rate to the degree of labor market tightness \( \sigma_u \). Indeed, if this condition is respected, the number of posted vacancies is equal to the one which would prevail in an economy where firms take into account that the vacancy filling rate decreases with the number of posted vacancies.

**Wage determination in the unionized sector** I assume that the unique concern of the trade unions is the welfare of their members, who are either employed or unemployed. At the time the wage rate is bargained over, \( \alpha^U - u^U_t \) members are employed and will attain the level of utility \( W^U_t \) at the end of the period, while \( u^U_t \) members are unemployed, out of which \( s^U_t u^U_t \) will form a
match and attain the level of utility $W^U_t$ at the end of the period, the remaining $(1 - s^U_t)u^U_t$ attaining the level of utility $U^U_t$. Consequently, the utility function of the unions takes the following form:

$$
\Omega_t = (\alpha^U - u^U_t)W^U_t + u^U_t [s^U_t W^U_t + (1 - s^U_t)U^U_t] \\
\Omega_t = n^U_t(W^U_t - U^U_t) + o_t \tag{13}
$$

where $o_t = \alpha^U U^U_t$ is the part of the union’s utility which does not depend on the wage rate.

This utilitarian specification presents several advantages. First, this specification, in keeping with MacDonald and Solow (1981) and Oswald (1982), is a common approach in the trade union literature (see Calmfors (1982), Sampson (1983), Kidd and Oswald (1987), Pissarides (1986) among others). Second, it allows a immediate comparison of the workers’ objectives across sectors. Indeed, while each worker in sector $N$ seeks to maximize the marginal workers’ surplus $W^N_t - U^N_t$, each union aims at maximizing the sum of the marginal workers’ surplus $n^U_t(W^U_t - U^U_t)$\(^5\). Both the workers and the unions foresee that their demands will affect the hiring decision of the firms. However, as described in Delacroix (2006) in a monopoly union model, the unique specification of the unionized sector is that unions internalize the negative impact of wages on employment when negotiating wages, because they do not only care about the workers’ surplus but also about the number of workers, $n^U_t$, enjoying this surplus. Third, the search and matching framework provides a natural way of modeling the workers’ fall back utility $U^U_t$, which is not fixed but fluctuates with the expected evolution of the degree of market tightness. Fourthly, the unions’ objective is directly derived from the members’ preferences, which is not the case with Stone-Geary utility forms. Moreover, the unions’ specification, as described by equation (13), allows for political considerations. Indeed, even if employment and wages have equal weight in the unions’ utility function, I will show later in this section that the relative importance of these issues is endogenous and varies along the cycle.

Unlike several recent papers studying trade unions and business cycles (Zanetti (2007a), Mattesini and Rossi (2007), Faia and Rossi (2009)) who adopt a monopoly union model, I assume that, at the beginning of each period, the unions bargain with the firms over the wages. Moreover, as mentioned earlier, firms decide subsequently and unilaterally the number of vacancies to post, based on the wage rate previously negotiated. This right-to-manage model, in line with Nickell (1982) and Nickell and Andrews (1983), allows me to keep the analysis as general as possible.\(^6\) The maximization program of this right-to-manage model is therefore the following:

\(^5\)This is because $\alpha^U U^U_t$ does not depend neither on $w^U_t$ nor on $n^U_t$.

\(^6\)Right-to-manage bargained wages are not Pareto-efficient. Efficient contracts can be obtained if firms and unions would bargained simultaneously over wages and employment, as shown by Leontief (1946). However, as argued by Calmfors and Horn (1986) and Oswald (1993), negotiations do generally not include employment explicitly.
\[
\max_{w_t^U} [n_t^U (W_t^U - U_t^U)]^{\eta_t^U} [n_t^U J_t^U]^{1 - \eta_t^U}
\]

s.t. JC curve: \( \frac{\kappa_t}{q_t} = x_t - w_t^U + E_t \beta (1 - \lambda_t) \frac{w_t}{q_{t+1}} \)

where \( \eta_t^U \) is the unions’ bargaining power. This collective Nash bargaining can be interpreted as follows. If the bargaining is successful and leads to an agreement on the wage, the firms benefit from \( n_t^U \) matches with average value \( J_t^U \). If the bargaining fails, the firms get a utility of zero. Concerning the unions, each of the \( n_t^U \) workers attain a level of utility \( W_t^U \) in case of agreement. The outside option is characterized by these \( n_t^U \) individuals becoming or staying unemployed.

It is important noting that the unique difference between the two sectors stands in the relevance of the hiring constraint, represented by the JC equation. One can observe that the Nash product of the collective bargaining is function of both the wage and the employment rate. The hiring constraint is therefore binding. The following mechanism is taken into account by the unions: the higher the wages, the lower the incentive for the firms to post vacancies, the lower the probability for unemployed workers to find a job, the lower the employment rate, the lower the total utility of the workers \( \Omega_t \).

\[ \Omega_t = \Omega_t(w_t^U, n_t^U(w_t^U)) \text{ with } \Omega'_w > 0, \Omega'_n > 0 \text{ and } n'_w < 0 \]

The negative correlation between the wage rate and the job finding rate embodies the trade-off faced by the union. This trade-off would disappear in case the hiring decision were taken before the wage bargaining. In this sense, the timing assumption is crucial in this sector. To see this, let us consider the alternative timing were vacancies are posted ex ante. With this timing, the hiring decision would be based on the expected, or promised, wage level. But unions can not credibly announce that they will moderate the wage rate in order to promote hirings. Indeed, once the vacancies have been posted, nothing prevent the unions to deviate from their announcement and pull the wage at the highest possible level. With this timing, the level of employment \( n_t^U \) would be taken as given. The maximization program would be reduced to the case of the wage bargaining of the average match, which is equal to the wage bargaining of the marginal match due to the constant returns to scale.\(^7\)

The FOC leads to the following equilibrium condition (see Appendix B.1):

\[ S_t^{wU} = \frac{\eta_t^U \sigma_u n_t^U}{(1 - \sigma_u) n_t^U + (1 - \eta_t^U) \sigma_u n_t^U} S_t^{fU} \]  

\(^7\)With constant return to scale, the firm’s employment value of the marginal and of the average match are equal, as well as the employment value of both the marginal and the average worker. Therefore, as argued by Gertler and Trigari (2009), the maximization program of the marginal match (when the wage is bargained over individually by each employee and employer) is equivalent to the maximization program of the average match (when the wage is bargained over collectively by each union and employer).
It follows that the wage curve in the unionized sector is (see Appendix B.2):

$$w^U_t = \tilde{\eta}_t \left[ x_t + E_t \beta (1 - \lambda^U) \kappa \theta_{t+1}^U \right] + (1 - \tilde{\eta}_t) b - \Theta_t$$  \hspace{1cm} (15)

where

$$\tilde{\eta}_t = \frac{\eta^U \sigma_u n^U_t}{\sigma_u n^U_t + (1 - \sigma_u) m^U_t} \leq \eta^U$$

$$\Theta_t = E_t \beta (1 - \lambda^U) \frac{K}{q_{t+1}} (1 - s^U_{t+1}) \frac{\tilde{\eta}_{t+1} - \tilde{\eta}_t}{1 - \tilde{\eta}_{t+1}}$$

Using $\tilde{\eta}_t$, I can rewrite (14) as:

$$\frac{S^w_t}{S^U_t} = \frac{\tilde{\eta}_t}{1 - \tilde{\eta}_t} \quad \frac{S^w_t}{S^U_t} = \tilde{\eta}_t$$

I define the concept of effective bargaining power of a agent as the share of the total surplus obtained by this agent. In the non unionized sector, bargaining power and effective bargaining power are two similar concepts: the workers’ bargaining power equals the proportion of the surplus going to the workers. In the unionized sector, these two concepts are different: the unions’ bargaining power $\eta^U$ does not equal the unions’ effective bargaining power $\tilde{\eta}_t$.

In order to get the intuition behind this result, I write the union’s effective bargaining power in this way:

$$\tilde{\eta}_t = \eta^U \left( \frac{1}{\sigma_u \eta^U m^U_t n^U_t + 1} \right) \in (\sigma_u \eta^U, \eta^U)$$  \hspace{1cm} (16)

When looking at equation (16), it comes out that the specificity of the union sector is materialized by the ratio $\frac{m^U_t}{n^U_t}$. More specifically, the unions’ effective bargaining power is equal to their bargaining power $\eta^U$ multiplied by a second term smaller than one and decreasing in $\frac{m^U_t}{n^U_t}$. This equation states that the unions’ effective bargaining power decreases with the proportion of newly employed workers over the employed workers (including the newly employed workers) within the union, because this category of employees have ambivalent preferences over wage and employment. Indeed, while the $\alpha^U - u^U_t$ employees at the beginning of the period benefit clearly from a high wage rate, the $m^U_t$ newly hired workers benefit from a moderate wage rate which make it easier to find a job but also from a high wage rate once they have been hired. The key is that,

8Notice that in the theoretical case where there were no unemployment at the beginning of the period (or more realistically, if unemployed workers were not taken into consideration by the unions), the ratio $\frac{m^U_t}{n^U_t}$ would be equal to zero and $\tilde{\eta}_t = \eta^U$. The result would be similar to an individual Nash bargaining.
even though the wage is negotiated ex ante on behalf of $\alpha^U - u^U_i$ employed workers and $u^U_i$ unemployed workers, the outcome of the bargaining (i.e the wage rate and the corresponding job finding probability) affects differently other categories of workers: the wage rate concerns ex post the $n^U_i = \alpha^U - u^U_i + m^U_i$ employed workers, and the corresponding job finding probability concerns the $m^U_i$ newly employed workers. The ratio $\frac{m^U_i}{n^U_i}$ represents therefore the power struggle among the union’s members between the ones who benefit from a high wage rate and the ones who benefit from a high job finding probability (and therefore from a moderate wage), keeping in mind that the newly hired workers belong to both categories because of their ambivalent preferences. This ratio introduces political considerations and reflects the internal conflict within the union. The higher the proportion of newly hired workers, the stronger the downwards pressure on the wage rate, the lower the unions’ power to drag a large part of the total surplus. Therefore, the composition of the union impacts the outcome of the wage negotiation and the ratio $\frac{m^U_i}{n^U_i}$ materializes this composition effect.

Consequently, I find the following salient result. The unions’ preferences over wages and employment, driven by the aforementioned composition effect, are endogenous and fluctuating with the situation on the labor market. Indeed, the ratio $\frac{m^U_i}{n^U_i}$ is increasing in $u^U_i$, the size of the pool of unemployed workers at the beginning of the period, and in $v^U_i$, the number of vacancies. Consequently, if the employment rate at the beginning of the period is high, the unemployment issue is played down and unions push the wages up in order to satisfy employed workers. If the pool of unemployed workers at the beginning of the period is big, unions moderate the wages in order to boost employment. This states that the degree of market tightness reflects on the composition of the unions and, through this channel, shapes the unions’ preferences over wages and employment.

Two comments are in order when considering the wage equation (15). First, like in the non unionized sector, the equilibrium wage in the unionized sector is a weighted average of two extreme wage levels: $x_t + E_t \beta(1 - \lambda^U) \kappa \theta^{U+1}_i + b$. Indeed, the third term in the right hand side in equation (15), $\Theta_t$, becomes negligible as the persistence of the model increases. Second, unlike in the non unionized sector, the sharing rule does not allocate a constant share of the bargaining set to the union and to the firm. Indeed, the weights, $\bar{\eta}_t$ and $(1 - \bar{\eta}_t)$, are not fixed but endogenous. The effective bargaining power, and therefore the wage rate, fluctuate according to the degree of market tightness. These remark leads to an important result. The fluctuations of the wage rate in the unionized sector stem from two different sources: the fluctuations of the total surplus and the fluctuations of the unions’ effective bargaining power.
3 Countercyclical union power

In this section, I seek to illustrate the mechanism through which wage rigidity arises when wages are collectively bargained. The degree of wage rigidity is assessed based on how the wage fluctuates when the economy is hit by a productivity shock.

3.1 Linearization

In order to better inspect the mechanism driving the effect of the productivity shock on the labor market, I consider a log-linear approximation of the model.

- (End of period) employment dynamics:
  \[ \hat{n}_t^N = -\lambda^N \hat{u}_t^N + \lambda^N \hat{m}_t^N \]
  \[ \hat{n}_t^U = -\lambda^U \hat{u}_t^U + \lambda^U \hat{m}_t^U \]

- (Beginning of period) unemployment dynamics:
  \[ \hat{u}_t^N = -s^N (1 - \lambda^N)\hat{n}_{t-1}^N \]
  \[ \hat{u}_t^U = -s^U (1 - \lambda^U)\hat{n}_{t-1}^U \]

Using these two equations and the Beveridge curve, I can write:

- Matching process:
  \[ \hat{m}_t^N = \sigma_u \hat{u}_t^N + (1 - \sigma_u)\hat{v}_t^N \]
  \[ \hat{m}_t^U = \sigma_u \hat{u}_t^U + (1 - \sigma_u)\hat{v}_t^U \]

- Vacancy filling rate:
  \[ \hat{q}_t^N = \hat{m}_t^N - \hat{v}_t^N \]
  \[ \hat{q}_t^U = \hat{m}_t^U - \hat{v}_t^U \]

- Job finding rate:
  \[ \hat{s}_t^N = \hat{m}_t^N - \hat{u}_t^N \]
  \[ \hat{s}_t^U = \hat{m}_t^U - \hat{u}_t^U \]

- Labor market tightness:
  \[ \hat{\theta}_t^N = \hat{v}_t^N - \hat{u}_t^N \]
  \[ \hat{\theta}_t^U = \hat{v}_t^U - \hat{u}_t^U \]

- JC curve:
  \[ \hat{\theta}_t^N = \phi^N \hat{x}_t - \phi^N \bar{w}^N \hat{u}_t^N + \beta(1 - \lambda^N)\hat{\theta}_{t+1}^N \]
  \[ \hat{\theta}_t^U = \phi^U \hat{x}_t - \phi^U \bar{w}^U \hat{u}_t^U + \beta(1 - \lambda^U)\hat{\theta}_{t+1}^U \]

where \( \bar{w}^i = \frac{w^i}{x^i}, \phi^i = \frac{1}{\pi_x K^i}, \) and \( K^i = \frac{\sigma}{\sigma_x^i}, i = N, U. \)
• Wage rate:

\[
\hat{w}_t^N = \hat{\eta}^N N \hat{x}_t + \hat{\eta}^N N \left[ \beta(1 - \lambda^N)N \hat{\kappa} \theta^N \right] \hat{\theta}^N t+1
\]

\[
\hat{w}_t^U = \hat{\eta}^U U \hat{x}_t + \hat{\eta}^U U \left[ \beta(1 - \lambda^U)U \hat{\kappa} \theta^U \right] \hat{\theta}^U t+1 + \hat{\eta}^U U \left[ \beta(1 - \lambda^U)U \hat{\kappa} \theta^U \right] \hat{\theta}^U t + \hat{O}_t
\]

where \( \hat{\eta}_t = \frac{1}{\hat{w}_t} \left[ (1 - \lambda^U)K^U (1 - s^U) \right] \left( \hat{\theta}_t - \hat{\theta}_{t+1} \right) \)

and \( \hat{\theta}_t = \hat{\theta}_t \), \( \hat{\kappa}_t = \hat{\kappa}_t \) and \( K^U = \frac{\hat{\kappa}_t}{\hat{q}_t^U} \).

• Union’s effective bargaining power:

\[
\hat{\eta}_t = - \left( \gamma^U (1 - \lambda^U + s^U) \right) \hat{\theta}_t^U - \left( \gamma^U (1 - \sigma_u) (1 - \lambda^U) \right) \hat{\theta}_t^U
\]

where \( \gamma^U = \frac{(1 - \sigma_u) \lambda^U}{(1 - \sigma_u) \lambda^U + \sigma_u} \).

3.2 Countercyclical effective bargaining power and wage rigidity

As evident from equation (19), a positive productivity shock has a negative impact on the unions’ effective bargaining power through its positive impact on the degree of labor market tightness \( \theta^U t \) (\( u_t^U \) being the beginning-of-period unemployment, it does not shift on impact). The intuition behind this result is that it becomes more valuable for the workers to be employed after the positive shock, because of the increase in the workers’ surplus. Therefore, union’s effective bargaining power decreases in order to foster employment through a moderate increase in the wage and in so doing to increase the number of workers benefiting from this greater surplus.

Consequently, the wage rate is driven by two elements: the procyclical surplus from the match and the countercyclical unions’ effective bargaining power. The second effect, absent in the non unionized sector, makes the wage rate more rigid, i.e. less responsive to the business cycle. It is worth noting that the nature of the wage rigidity arising here contrasts with the one presented in Shimer (2004), Hall (2005) or Gertler and Trigari (2009). In these models, wages do not respond\(^9\) to current productivity. In the present paper, the labor market is a spot market, past labor market conditions do not determine current wages, and wages are function of the current productivity level. The key of the wage rigidity mechanism lies in the countercyclicality of one component of the wage, the unions’ effective bargaining power, which dampens wage fluctuations.

\(^9\)In Gertler and Trigari (2009) this unresponsiveness of the wage only concerns a certain proportion of the firms.
4 Quantitative assessment of the model

In this section the quantitative properties of the model are investigated by studying the impulse response of the labor market to a positive productivity shock in both sectors.

4.1 Calibration

Table 1: Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic process for labor productivity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>$\rho$</td>
<td>0.98</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>$\sigma$</td>
<td>0.008</td>
</tr>
<tr>
<td>Common parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
<td>0.991/3</td>
</tr>
<tr>
<td>Elasticity of $m$ with respect to $u$</td>
<td>$\sigma_m$</td>
<td>0.5</td>
</tr>
<tr>
<td>Unemployment income</td>
<td>$\bar{b}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Efficiency of the matching process</td>
<td>$\sigma_m$</td>
<td>0.6364 set to target $\theta^N = 0.5$</td>
</tr>
<tr>
<td>Vacancy posting cost</td>
<td>$\kappa$</td>
<td>0.3553 set to target $s^N = 0.45$</td>
</tr>
<tr>
<td>Non unionized sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separation rate</td>
<td>$\lambda^N$</td>
<td>0.1/3</td>
</tr>
<tr>
<td>Workers’ bargaining power</td>
<td>$\eta^N$</td>
<td>0.5</td>
</tr>
<tr>
<td>Unionized sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separation rate</td>
<td>$\lambda^U$</td>
<td>0.1/3</td>
</tr>
<tr>
<td>Workers’ bargaining power</td>
<td>$\eta^U$</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The calibration of the model is described in table 1. These values are chosen to match the empirical regularities of U.S.

I interpret a period as a month. The discount factor is set to 0.991/3 which corresponds to a yearly interest rate of 4% commonly used in the macro-RBC literature.

The log productivity level $x_t$ is assumed to follow an AR(1) process: $\log(x_t) = \rho \log(x_{t-1}) + \epsilon_t$ where $\epsilon \sim N(0,\sigma^2)$. The persistence of the technology shock is set to $\rho = 0.98$ and the standard deviation to $\sigma = 0.008$. This standard calibration is used by Rogerson and Shimer (2010) and is based on the estimations of Cooley and Prescott (1995). The mean of $x$ is normalized to one.

I target the probability $s^N$ that an unemployed worker forms a match within the period to 45%, implying unemployment spells of around two months. This
choice is consistent with Hall (2005) who estimates an monthly job finding rate of 0.48% and in line with the measure of this rate presented by Rogerson and Shimer (2010) for the US for the period 1948-2009. Each match has a probability to end $\lambda^N$ set to 0,1/3. This value is comprised within the broadly accepted range of 8% – 10% proposed by Hall (2005) and is similar to Shimer (2005) who measures this exit probability at 0,1/3 in average in the US. I target the degree of labor market tightness $\theta^N$ to 0.5, which is consistent with the estimate of 0.539 obtained by Hall (2005).

Considering the matching process, two parameters have to be discussed. First, the weight on unemployment $\sigma_u$, which represents the elasticity of the matches with respect to unemployment but also the elasticity of the vacancy filling rate with respect to the labor market tightness, is set equal to 0.5. This value is consistent with the range $[0.5 - 0.7]$ proposed by Burda and Wyplosz (1994) based on estimations of the matching function for some western European countries. Second, $\sigma_m$ is obtained from steady state calculations.

Following the literature, I set the value of $\eta^N$ to 0.5 to satisfy the Hosios condition and therefore to obtain an efficient decentralized equilibrium. This value is suggested by Mortensen (1994) and Mortensen and Pissarides (1994) for reasons of symmetry.

In contrast with the other parameters and targets, there exists a debate about the value of non work activity $\bar{b} = b/x$, revived by the recent paper by Hagedorn and Manovskii (2008) which proposes a new estimate of this value at 0.95. Indeed, unlike Shimer (2005) who restricts the value of non work activity to the unemployment benefits and sets $\bar{b}$ equal to 0.4, Hagedorn and Manovskii (2008) additionally integrate the home production and the value of leisure. Delacroix (2006) also distinguishes within the unemployment income set at 0.6 a home production of 0.3 and unemployment benefits of 0.3, the value of $\bar{b}$ is set at 0.6. In order to keep my results as plausible as possible, I choose an average value of 0.8.

If the same effective bargaining power were applying for the workers in the non unionized sector and for the unions in the unionized sector, the steady state values would be equal in both sectors. This condition can be written as the following: $\eta^U = \bar{\eta}^U$ s.t. $\bar{\eta}^U = 0.5$.

That the presence of unions increases workers’ bargaining power is beyond dispute. The largely documented union wage premium reflects this difference in bargaining power. This observation requires to fix the unions’ effective bargaining power $\bar{\eta}$ greater than the workers’ bargaining power $\eta^N$, or equivalently to set $\eta^U > \bar{\eta}$. In the search and matching literature, the value of the workers bargaining power is generally set at 0.5 for symmetry reason because of a lack of evidence. There is not more evidence for the value of the unions’ bargaining power $\eta^U$ beside the observation that it lies in the interval $[\bar{\eta}, 1]$. I set

\[\eta^U = 0.8\]
\(\eta^U = 0.9\) in the baseline calibration and propose in the last section to check the implications of alternative values (\(\eta^U = \bar{\eta}^U\), \(\eta^U = 0.7\) and \(\eta^U = 1\)) for the model’s dynamics.

In order to ease the comparison between the two sector and to identify the specificity of the collective wage bargaining, the separation rate in the unionized sector is set equal to the one prevailing in the non unionized sector. A robustness check with an alternative calibration for \(\lambda^U\) is provided in the last section.

The steady state of the model is shown in table 2. In the non unionized sector, the steady state beginning-of-period unemployment rate \(u\) is at 7.1\%. The same rate in 10 percentage points higher in the unionized sector. This result is quantitatively in line with the one proposed by Delacroix (2006). The high level of the wage rate in the unionized sector decreases the firms’ value of employment and restrains the vacancy postings. As a result, the unemployment rate is higher and the unemployment duration is larger.

Table 2: Steady State

<table>
<thead>
<tr>
<th></th>
<th>Sector N</th>
<th>Sector U</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>0.985</td>
<td>0.995</td>
</tr>
<tr>
<td>Vacancies</td>
<td>0.036</td>
<td>0.012</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>7.1%</td>
<td>17.1%</td>
</tr>
<tr>
<td>Unemployment duration</td>
<td>2.2m.</td>
<td>6m.</td>
</tr>
</tbody>
</table>

4.2 Dynamics

In this section, I first study the business cycle properties of the model with the baseline calibration. In a next step, I check the robustness of the results for alternative values of \(\eta^U\) and \(\lambda^U\).

4.2.1 Impulse responses to productivity

Figure 1 shows the response of the labor market to a positive productivity shock of one standard deviation. In order to disentangle the effect on the labor market dynamics of, in one hand, the difference in steady state (called steady state effect) and, in the other hand, of the volatility of the effective bargaining power (called bargaining power effect), I show with the dotted line the dynamics of a non unionized sector artificially pushed at the steady state level of the unionized sector. This artificial case is referred to as the intermediate sector. The difference between the dashed line (unionized sector) and the dotted line only stems from the volatility of the effective bargaining power. The difference between the plain line (non unionized sector) and the dotted line arises from the difference in steady state between the two sectors.
Let us first compare the unionized sector with the intermediate one. The key of the mechanism lies in the countercyclical unions’ effective bargaining power, which contrasts with the fixed workers’ bargaining power in the non-unionized sector. On impact, its decrease slightly dampens the volatility of the wage (the dashed line lies below the dotted line), enough to create an extra surplus for the firms which react in posting more vacancies. Relatedly, employment reacts more strongly. As employment goes up, more employees call for a high level of wage, which results in unions being more “wage-oriented”. The unions’ effective bargaining power increases, slowing down the returning of the wage towards its steady state. The firms’ surplus, as well as the vacancies and the employment rate, decreases relatively sharply.

Second, I compare the non-unionized sector with the intermediate one. It is interesting to notice that even though the wage responds more in the intermediate sector, employment reacts also more. The following reasoning explains this result. As argued by Hagedorn and Manovskii (2008), what gives the incentive to firms to post vacancies is the size of the percentage changes of the firms’ surplus in response to a shock. These percentage changes are bigger the smaller the firms’ surplus. In the intermediate sector, the workers’ bargaining power and therefore the wage are higher. The firms’ surplus is smaller. Moreover, the vacancy duration is lower the higher the unemployment rate, which leads to a lower saving of hiring cost in case of match in the intermediate sector. This decreases the total surplus and, the firms’ surplus being a fixed part of the total surplus, the firms’ surplus. As a result, the percentage changes of the firms’ surplus are bigger in the intermediate case, and so is the firms’ incentive to post vacancies.

4.2.2 Robustness

Analyzing the dynamic properties of the model using different values for $\eta^U$ and $\lambda^U$ can be seen as a robustness check.

**The value of $\eta^U$**. Given the lack of evidence to pin down the value of $\eta^U$, I propose to analyze the response of the labor market for 4 values of $\eta^U$: $\eta^U = \bar{\eta}^U$ (value for which the steady states of both sectors are equalized), $\eta^U = 0.7$, $\eta^U = 0.9$ (baseline calibration) and $\eta^U = 1$ (monopoly union). Figure 2 shows how the (workers and unions’) effective bargaining power, the wage and the employment rate respond to the positive productivity shock. An interesting aspect of the model is that the quantitative effect of the volatility of the effective bargaining power becomes larger the higher the unions’ bargaining power. Indeed, if this effect is negligible for low values of $\eta^U$, it is striking in the monopoly union case. The reason for that can be seen in equation (17). The change in the effective bargaining power $\hat{\eta}_t$ has a higher impact on the wage the higher the steady state value $\hat{\eta}$ and therefore the higher $\eta^U$. Moreover, the greater the change in the wage, the greater the impact on the labor market tightness $\theta^U$, the bigger the change in the unions’ effective bargaining power. Intuitively, as $\eta^U$ goes up, the workers’ surplus raises and unions moderate the wage increase in order to
boost employment so that a higher proportion of workers can benefit from this surplus.

**The value of $\lambda^U$** One could argue that the separation rate is lower in the unionized sector. The empirical literature proposes some evidence for this phenomenon. For example, Freeman (1980) shows that, in the US, tenure is greater for workers covered by union contracts and that the probability for their match to end is lower. Knight and Latreille (2000) and Antcliff and Saundry (2009) find similar results in the UK. To be in line with this strand of research, I consider the case where $\lambda^U = 0.08/3 < \lambda^N = 0.1/3$ and compare the results with the ones obtained in the previous section.

As shown in figure 3, the bargaining power effect is not substantially modified by the alternative calibration. The decrease of $\lambda^U$ dampens the volatility of the effective bargaining power but the overall effect on the wage and employment dynamics is small. However, the alternative calibration modifies the steady state effect. Hirings increase less than with the baseline calibration, because the firms benefit from a bigger surplus when the exit rate is low and this leads to smaller percentage changes of the firms’ surplus. By the aforementioned mechanism, firm’s incentive to post vacancies is lower, with explains the moderate increase in employment.

## 5 Concluding remarks

By modeling labor union in a search and matching framework, I develop a tractable model of the labor market in which the bargaining power of the union is analyzed. I show that the unions’ effective bargaining power fluctuates countercyclically, creating a second source of union wage volatility. When hit by a positive productivity shock, the union reacts in moderating the wage increase. This is due to the impact of the shock on the composition of the union. I am therefore able to explain the origin of the wage rigidity, which leads to an amplification of the employment response.

This paper chooses a simplistic framework in order to give a good intuition on how labor unions affect the labor market. This explains my choice of focusing only on the labor market. The possible extension of the model, which can be addressed in future research, is therefore to introduce the pricing policy of firms when allowing for interaction between the two sectors, which would lead to a study of the role of labor union in shaping the volatility of inflation.

To conclude, the model presented in the present paper represents an improvement over the current literature by bringing together two strands of research. It develops our theoretical understanding of the source of wage and employment volatility as well as the role of labor unions in shaping these volatilities.
References


A Alternative timing in the non unionized sector

In this appendix, I drop the superscript $N$ to simplify the notation.

Let us consider an alternative timing in the non unionized sector in order to show that both timings lead to the same equilibrium. In this appendix, I make the assumption that firms decide first how many vacancies to post and, once the matching process has taken place, wages are bargained over. I solve the model backward, starting by the wage negotiation. See Cahuc and Wasmer (2001) for a detailed analysis.

Wage negotiation The maximization program is the following:

$$\max_{w_t} [W_t - U_t]^{\eta}[J_t]^{1-\eta}$$

The job creation curve does not constitute a constraint for the wage negotiation given that the level of employment has already been decided.

The first order condition states that the link between the worker and firm’s surpluses is:

$$\frac{S_t^w}{S_t^f} = \frac{\eta}{1-\eta}$$

Hiring decision The wage curve, indicating the level of the wage for any level of employment, is known by the firms at the beginning of the period. Therefore, with this timing, they have the possibility to manipulate the wage through their employment policy. The number of posted vacancies results from the following profit’s maximization process:

$$\max_{n_t,v_t} \pi_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [x_t n_t - w_t n_t - \kappa v_t]$$

s.t. \begin{align*}
    n_t &= (1-\lambda) n_{t-1} + q_t v_t \\
    \text{Wage curve: } w_t &= w_t(n_t)
\end{align*}

The first order conditions with respect to $n_t$ and $v_t$ are given by:

$$x_t - w_t - \frac{\partial w_t}{\partial n_t} n_t - \mu_t + E_t \beta \mu_{t+1} (1-\lambda) = 0$$

$$\mu_t = \frac{\kappa}{q_t}$$

where the Lagrangian multiplier $\mu_t$ represents the marginal value of employment.

Rearranging the first order conditions I obtain the job creation equation (JC):

$$\frac{\kappa}{q_t} = x_t - w_t - \frac{\partial w_t}{\partial n_t} n_t + E_t \beta (1-\lambda) \frac{\kappa}{q_{t+1}}$$

(21)
Equilibrium  Plugging (21) into (20), and using the expressions of $S_w^t$ and $S_f^t$, I obtain the following wage curve:

$$w^N_t = \eta^N [x_t - \frac{\partial w_t}{\partial m_t} n_t + E_t \beta (1 - \lambda^N) \kappa \theta^N_{t+1}] + (1 - \eta^N) b$$

(22)

The equilibrium obtained with this alternative timing differs from the one presented in the body of the paper by the extra element $\frac{\partial w_t}{\partial m_t} n_t$. This intra-firm bargaining effect is explained by the fact that the wage of each employee is renegotiated each time the scale of the firm changes (because of hirings or match endings). The firms takes therefore into account the fact that the employment level has an impact on the wage rate.

However, in case of constant return to scale, $\frac{\partial w_t}{\partial m_t}$ is equal to zero and the intra-firm bargaining effect vanishes. Equation (22) reduces to:

$$w^N_t = \eta^N [x_t + E_t \beta (1 - \lambda^N) \kappa \theta^N_{t+1}] + (1 - \eta^N) b$$

which is equivalent to (12).

B  Equilibrium wage in the unionized sector

B.1 Link between the worker and the firm’s surplus in the unionized sector

The maximization program of the union is the following (here again, I drop the superscript $i = U$ to simplify the notation but keep in mind that we are in the unionized sector):

$$\max_{n_t} [n_t J_t]^{1-\eta} [n_t (W_t - U_t)]^\eta$$

s.t JC equation: $\frac{\kappa}{q_t} = x_t - w_t + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}}$

FOC:

$$n_t J_t^{1-\eta} (W_t - U_t)^{\eta} + n_t (1-\eta) J_t^{-\eta} (W_t - U_t)^{\eta} \frac{\partial J_t}{\partial w_t} + n_t \eta J_t^{1-\eta} (W_t - U_t)^{\eta-1} \frac{\partial W_t}{\partial w_t} = 0$$

$$-n_t' = -n_t (1-\eta) J_t^{-1} + n_t \eta (W_t - U_t)^{-1}$$

It comes from the definition of $n_t = 1 - u_t + m_t$ that:

$$n_t' = \sigma_m (1 - \sigma_u) w_t \theta_t^{-\sigma} \theta_{t+w}$$
Moreover, from the JC and the definition of \( q_t \) I get:

\[
\theta_t = \left[ \frac{\sigma_m}{\kappa} \left( x_t - w_t + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} \right) \right]^{\frac{1}{\sigma_u}}
\]

Therefore:

\[
\theta_{t,w}' = -\frac{\sigma_m}{\sigma_u \kappa} \theta_t^{1 - \sigma_u}
\]

Plugging the expression of \( \theta_{t,w}' \) in the expression of \( n_{t,w}' \), I obtain:

\[
n_{t,w}' = -\frac{(1 - \sigma_u)m_t}{\sigma_u n_t}
\]

Using this expression, the FOC can be rewritten:

\[
(W_t - U_t) \left( \frac{1 - \eta}{\eta} + \frac{1 - \sigma_u m_t}{\sigma_u n_t} \frac{1}{\eta} \right) = J_t
\]

\[
W_t - U_t = \frac{\eta \sigma_u n_t}{(1 - \sigma_u)m_t + (1 - \eta)\sigma_u n_t} J_t
\]

which is equivalent to (14).

### B.2 Wage curve in the unionized sector

For convenience I omit again the superscript \( i = U \).

From (9) and (10), we have:

\[
W_t - U_t = w_t - b + E_t(1 - \lambda)\beta (1 - s_{t+1})(W_{t+1} - U_{t+1})
\]

Using the condition (14) we can rewrite this equation as:

\[
\frac{\eta \sigma_u n_t}{(1 - \sigma_u)m_t + (1 - \eta)\sigma_u n_t} J_t = w_t - b + E_t(1 - \lambda)\beta (1 - s_{t+1}) \frac{\kappa}{q_{t+1}}
\]

Plugging the value of \( J_t = \frac{\sigma_u}{q_{t+1}} = x_t - w_t + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} \) into this equation:

\[
\frac{\eta \sigma_u n_t}{(1 - \sigma_u)m_t + (1 - \eta)\sigma_u n_t} (x_t - w_t + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}}) = w_t - b + E_t(1 - \lambda)\beta (1 - s_{t+1}) \frac{\eta \sigma_u n_{t+1}}{(1 - \sigma_u)m_{t+1} + (1 - \eta)\sigma_u n_{t+1}} \frac{\kappa}{q_{t+1}}
\]

Rearranging leads to:

\[
w_t = \frac{\eta \sigma_u n_t}{(1 - \sigma_u)m_t + \sigma_u n_t} \left[ x_t + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} \right]
\]

\[
+ \frac{(1 - \sigma_u)m_t + (1 - \eta)\sigma_u n_t}{(1 - \sigma_u)m_t + \sigma_u n_t} \left[ b - E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} (1 - s_{t+1}) \right] \frac{\eta \sigma_u n_{t+1}}{(1 - \sigma_u)m_{t+1} + (1 - \eta)\sigma_u n_{t+1}}
\]

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\[ w_t = \tilde{\eta}_t \left[ x_t + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} \right] + (1 - \tilde{\eta}_t) \left[ b - E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} (1 - s_{t+1}) \frac{\tilde{\eta}_{t+1}}{1 - \tilde{\eta}_{t+1}} \right] \]

with \( \tilde{\eta}_t = \frac{\eta \sigma_u n_t}{(1 - \sigma_u) n_t + \sigma_u n_t} \).

\[ w_t = \tilde{\eta}_t \left[ x_t + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} q_{t+1} + E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} (1 - s_{t+1}) \right] + (1 - \tilde{\eta}_t) b - (1 - \tilde{\eta}_t) \left[ E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} (1 - s_{t+1}) \frac{\tilde{\eta}_{t+1}}{1 - \tilde{\eta}_{t+1}} \right] \]

\[ w_t = \tilde{\eta}_t \left[ x_t + E_t \beta (1 - \lambda) \kappa \theta_{t+1} \right] + (1 - \tilde{\eta}_t) b - \left[ E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} (1 - s_{t+1}) \right] \left[ -\tilde{\eta}_t + \frac{\tilde{\eta}_{t+1} (1 - \tilde{\eta}_t)}{1 - \tilde{\eta}_{t+1}} \right] \]

\[ w_t = \tilde{\eta}_t \left[ x_t + E_t \beta (1 - \lambda) \kappa \theta_{t+1} \right] + (1 - \tilde{\eta}_t) b - \left[ E_t \beta (1 - \lambda) \frac{\kappa}{q_{t+1}} (1 - s_{t+1}) \right] \left[ \frac{\tilde{\eta}_{t+1} - \tilde{\eta}_t}{1 - \tilde{\eta}_{t+1}} \right] \]

This is equivalent to (15).
Figure 1: Impulse responses to a positive productivity shock

Note: Percentage deviation from the steady state following a positive productivity shock of one standard deviation.
Figure 2: Impulse responses to a positive productivity shock

Note: Percentage deviation from the steady state following a positive productivity shock of one standard deviation.
Figure 3: Impulse responses to a positive productivity shock

Note: Percentage deviation from the steady state following a positive productivity shock of one standard deviation.