The Macroeconomics of Credit Market Imperfections (Part II): Dynamic Models

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Reading Group: Topics of Macroeconomics (SS08)
Outline

Motivation
  Taking stock
  From static to dynamic

Going Dynamic: The Prototype Model
  Settings
  Constraints and equilibrium conditions
  Solution for the baseline case

Credit Traps and Credit Cycles
  Endogenous technological change
  Introducing investment output heterogeneities
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What have we done last time? Static models

- Modelling credit market imperfections. Key factors: $\lambda$ – measure of imperfection; $\omega$ – (exogenous) net worth;
  - Market economy fails to allocate the credit to its most productive use;
  - Net worth / balance sheet conditions (for both lenders and borrowers) play crucial roles in allocating the credit.
- Partial equilibrium models with homo- / heterogeneous agents;
- General equilibrium models with open economy extensions.

To be discussed this time: Dynamic models on dynamic feedback from the investment to the net worth — Persistence, volatility, and growth.
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Credit market imperfections in dynamic macro

- Key difference in dynamic models: Endogenized $\omega$ — Dynamic feedback from the investment to the net worth.

- Credit market imperfections in dynamic macro: Standard view
  - Bernanke-Gertler (BG, 1989): (Moral hazard based) financial accelerator
    - Higher borrowers’ net worth $\rightarrow$ Less credit frictions $\rightarrow$ Higher investment $\rightarrow$ Higher borrower net worth

- Matsuyama: Beyond BG
  - Balance sheet effects from both lenders and borrowers;
  - Credit channels in growth: Both volume and composition of the credit matter. Much richer patterns.
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Augmented *OLG* model. Infinite, discrete time horizon $T = 0, 1, \ldots$

- A unit measure of homogeneous agents.
  - 1st period, supply one unit of labor, and save wage $S_t = w_t L_t = w(k_t)$. Then choose to be entrepreneurs / lenders, starting investment projects;
  - 2nd period, production, consume.

- Final good $Y_t$, for either consumption or investment. Produced by NC technology $Y_t = F(K_t, L_t)$, and
  $$y_t = \frac{Y_t}{L_t} = F \left( \frac{K_t}{L_t}, 1 \right) = f(k_t), \text{ with } f' > 0 > f'';$$

- Competitive factor market
  - Rental rate of capital $\rho_t = f'(k_t);$  
  - Wage rate of labor $w_t = f(k_t) - k_t f'(k_t).$
Generations are linked via production. Implication: A rise in productivity increases the net worth of next generation.
An entrepreneur can choose one (and only one) from \( j \in \{1, ..., J\} \) indivisible investment projects. Investment technology

<table>
<thead>
<tr>
<th>Period ( t )</th>
<th>Period ( t + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project ( j )</td>
<td>( m_j ) final good</td>
</tr>
</tbody>
</table>

- \( m_j \): Measure of indivisibility – Fixed set-up cost;
- \( R_j \): Project productivity in capital. Then transformed into final goods by \( Y_t = F(K_t, L_t) \), combined with labor provided by the young;
- \( B_j \): Project productivity in final goods;
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Payoffs and constraints

- **Payoffs of agents:**
  - Entrepreneur $j$: $C^j_t = m_j R_j \rho_{t+1} + m_j B_j - r_{t+1} (m_j - w_j)$;
  - A lender: $C^o_t = r_{t+1} w_t$.

- **$PC - j$:** One is willing to be entrepreneur of project $j$ only if $C^j_t \geq C^o_t \iff R_j f'(k_{t+1}) + B_j \geq r_{t+1}$ — In the perfect credit market, all credit goes to the projects with highest $PC - j$.

- **$BC - j$:** $\lambda_j m_j R_j f'(k_{t+1}) + \mu_j m_j B_j \geq r_{t+1} [m_j - w(k_t)]$. Agency problems in two dimensions:
  - $\lambda_j$ on capital production during project $j$;
  - $\mu_j$ on final goods production during project $j$. 
Payoffs and constraints (cont’d)

- $PC - j$ and $BC - j$:

\[
\frac{1}{r_{t+1}} = \max \left\{ \frac{1 - \frac{w(k_t)}{m_j}}{\lambda_j R_j f'(k_{t+1}) + \mu_j B_j}, \frac{1}{R_j f'(k_{t+1}) + B_j} \right\}
\]

Entrepreneur:

Lender:

$\text{lender}(t+1)$ for $t$

Work with $m_j$, earn $w(k_t)$

Generation $t$

Generation $t-1$

Generation $t+1$

Entrepreneur: $m_j$

Lender: $w(k_t)$
Equilibrium conditions

- (1) Resource constraint in investment \((X_{jt} - \text{measure of } j \text{ projects})\):

\[ w(k_t) \geq \sum_j (m_j X_{jt}); \]

- (2) Resource constraint in capital input:

\[ k_{t+1} \leq \sum_j (m_j R_j X_{jt}); \]

- (3) \(PC - j\) and \(BC - j\):

\[ \frac{1}{r_{t+1}} \leq \max \left\{ \frac{1 - \frac{w(k_t)}{m_j}}{\lambda_j R_j f'(k_{t+1}) + \mu_j B_j}, \frac{1}{R_j f'(k_{t+1}) + B_j} \right\}. \]
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Convergence in the baseline case: $J = 1$

- Now have a look at the simplest case: Homogeneous investment projects, $J = 1$.

- Equilibrium conditions become
  - (1) Resource constraint in investment: $w(k_t) = mX_t$;
  - (2) Resource constraint in capital input:
    \[ k_{t+1} = mRX_t = Rw(k_t); \]
  - (3) $PC - j$ and $BC - j$:
    \[ r_{t+1} = \min \left\{ \frac{\lambda Rf'(Rw(k_t)) + \mu B}{1 - \frac{w(k_t)}{m}}, Rf'(Rw(k_t)) + B \right\}. \]

- Monotone convergence achieved under the assumptions:
  - Decreasing marginal return: $\partial \left( \frac{w(k)}{k} \right) / \partial k < 0$;
  - Inada conditions: $\lim_{k \to 0} \frac{w(k)}{k} = +\infty$; $\lim_{k \to +\infty} \frac{w(k)}{k} = 0$. 
Convergence in the baseline case (cont’d)

- Same as prototype \textit{OLG} models (no wonder):

\begin{equation}
\begin{aligned}
\text{Entrepreneur: } & \mathbf{z}_t, \text{ earn } \mathbf{w}_t \\
\text{Lender: } & \mathbf{w}_{t-1} \text{ for } \mathbf{z}_t \\
\vdots & \\
\text{Work with } & \mathbf{z}_t, \text{ earn } \mathbf{w}_t \\
\end{aligned}
\end{equation}
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Now introduce heterogeneity in productivity. Set $j \in \{1, \ldots, J\}$ and $B_j = 0$, i.e. project $j$ only returns $m_j R_j$ capital.

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<td>Project $j$</td>
<td>$m_j$ final good</td>
<td>$m_j R_j$ capital</td>
</tr>
</tbody>
</table>

Equilibrium conditions:

1. Resource constraint in investment: $w(k_t) = \sum_j (m_j X_{jt})$;
2. Resource constraint in capital input: $k_{t+1} = \sum_j (m_j R_j X_{jt})$;
3. $PC - j$ and $BC - j$: $\frac{r_{t+1}}{f'(k_{t+1})} \geq \max \left\{ \frac{R_j}{1 - \frac{w(k_t)}{m_j \lambda_j}}, 1 \right\}$.
Endogenous technological change (cont’d)
Pro-cyclical technological change

- Set $J = 2$ and productivity change $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$. Trade-off: Productivity versus agency problem – 2 may be subject to bigger agency problems than some mundane project 1 that use well-established technologies.
Pro-cyclical technological change: Convergence

- Exhibits various patterns of pro-cyclical convergence...

Credit-constrained growth

Credit traps

Credit collapse

$k_c < k^* < k^{**}$

Credit-constrained growth

$k^* < k_c < k^{**}$

Credit traps

$k^* < k_c < k^{**}$

Credit collapse
Pro-cyclical technological change: Intuition

- Driving forces: Rate of return, resource and credit constraints – Feed-back effects.

- For higher $k$
  - Borrowers can pledge more with higher net worth;
  - Credit composition may shift towards more productive projects;
  - However, capital deepening effect – The credit friction prevents the credit from flowing into the more productive project, for $k$ not high enough.

- For improved $\lambda$:
  - A higher $\lambda_1$ can make things worse by increasing $k_c$, thereby creating a credit trap or causing a credit collapse;
  - A higher $\lambda_2$ seems desired, sometimes.
Counter-cyclical technological change

- Set $J = 2$ and productivity change $R_2 > R_1 > \lambda_2 R_2 > \lambda_1 R_1$, $\frac{m_2}{m_1} > \frac{1-\lambda_1}{1-\lambda_2} \frac{R_2}{R_1}$. Trade-offs: (1) 1 is less productive with more agency problem; (2) 1 requires the smaller set-up cost.
Counter-cyclical technological change (cont’d)

- Exhibits various patterns of counter-cyclical convergence...

\[ k_{t+1} \]

- \[ k_c < k^* < k_{cc} < k^{**} \]
  - Take-over

- \[ k^* < k_c < k^{**} < k_{cc} \]
  - Credit cycles

- \[ k^* < k_c < k_{cc} < k^{**} \]
  - Credit cycle with trap
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Introducing heterogeneities in output of investment

Set $J = 2$, $R_1 = R$ and $B_1 = 0$; $R_2 = 0$ and $B_2 = B$, $f'(0) = +\infty$.

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<td>Project 1 (G)</td>
<td>$m_1$ final good</td>
<td>$m_1 R$ capital</td>
</tr>
<tr>
<td>Project 2 (B)</td>
<td>$m_2$ final good</td>
<td>$m_2 B$ final good</td>
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Implications of $G$ and $B$

- $G$ produces inputs that are complementary with labor inputs endowed by the next generation, helping to improve the net worth of the next generation;
- $B$ produces the good that can be directly consumed. Does not help to improve the net worth of the next generation.
Equilibrium conditions

Again, equilibrium conditions

- (1) Resource constraint in investment:
  \[ W(k_t) = m_1 X_{1t} + m_2 X_{2t}; \]

- (2) Resource constraint in capital input: \( k_{t+1} = m_1 R X_{1t}; \)

- (3) Credit constraints \( PC \) and \( BC \):
  \[
  f'(k_{t+1}) = \frac{R \max \left\{ \frac{1 - \frac{w(k_t)}{m_1}}{\lambda_1}, 1 \right\}}{B \max \left\{ \frac{1 - \frac{w(k_t)}{m_2}}{\mu_2}, 1 \right\}} \quad \text{when } X_{2t} > 0; \quad k_{t+1} = RW(k_t)
  \]
  \[
  \quad \text{when } X_{2t} = 0.
  \]
Baseline case: Prefect credit market

- Set $\lambda_1 = 1$, $\mu_2 = 1$, then
Baseline case: Prefect credit market (cont’d)
Credit market imperfection in projects $G$

- Set $\lambda_1 < 1$, $\mu_2 = 1$, then
  - Dynamic resource constraint: $k_{t+1} \leq R w(k_t)$;
  - $PC$ and $BC$ attached to projects $G$:

$$R f'(k_{t+1}) = B \max \left\{ \frac{1 - \frac{w(k_t)}{m_1}}{\lambda_1}, 1 \right\} \geq B.$$
Credit market imperfection in projects $G$ (cont’d)

- Exhibits various patterns of convergence with different parameters, not a big wonder:

$$
(f')^{-1}\left(\frac{B}{R}\right) = Rw(k_c)
$$

Echo effect  Slow recovery from recession  Multiple steady state
Credit market imperfection in projects $B$

- Set $\lambda_1 = 1$, $\mu_2 < 1$, then
  - Dynamic resource constraint: $k_{t+1} \leq Rw(k_t)$;
  - $PC$ and $BC$ attached to projects $B$:
    
    $$B = Rf'(k_{t+1}) \max \left\{ \frac{1 - \frac{w(k_t)}{m^2}}{\mu_2}, 1 \right\} \geq Rf'(k_{t+1}).$$
Credit market imperfection in projects $B$ (cont’d)

- **Pattern 1** Small $B$ or $\mu$: Projects $B$ never financed
Credit market imperfection in projects $B$ (cont’d)

- **Pattern 2** High $B$ and $\mu$: Overshooting
Pattern 3 Smaller $\mu$: Oscillatory convergence
Pattern 4 Even smaller $\mu$: Endogenous fluctuations, chaos
Credit market imperfection in projects $B$ (cont’d)

- Rich patterns: Neoclassical convergence – Endogenous fluctuations – Chaos. Intuition:
  - If projects $B$ suffer from major agency problems (a small $\mu$), they are never financed. All the credit always goes to $G$;
  - If projects $B$ suffer from minor agency problems (a large $\mu$), they are financed as soon as they become more profitable than projects $G$;
  - Fluctuations occur when agency problems with projects $B$ are too big to be financed when the net worth is low, but small enough to be financed when the net worth is high.

- Introducing non-monotonicity in macro dynamics.
- Credit market imperfections play crucial roles in amplifying business cycle fluctuations.
Summary

- What have we done so far?
  - OLG based dynamic models on dynamic feedback from the investment to the net worth;
  - Endogenous technical change via credit channels: Take-over, credit traps, credit collapse;
  - Heterogeneities in output of investments: Convergence, endogenous fluctuations, chaos.

- Flexible, tractable, compact model. Ready for more extensions:
  - More complicated constraints;
  - Heterogeneities in agents (inequality and poverty) and commodities (domestic and foreign goods), open economy, etc.
Matsuyama, K.  
Financial market globalization, symmetry-breaking, and endogenous inequality of nations.  

Matsuyama, K.  
Credit traps and credit cycles.  

Matsuyama, K.  
Aggregate implications of credit market imperfections.  
in D. Acemoglu, K. Rogoff, and M. Woodford. (eds.)  