I am grateful to Mike Woodford for helpful discussions and comments and to Roger Meservey for professional editing. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

© 2009 by Pierpaolo Benigno. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.
ABSTRACT

A simple New-Keynesian model is set out with AS-AD graphical analysis. The model is consistent with modern central banking, which targets short-term nominal interest rates instead of money supply aggregates. This simple framework enables us to analyze the economic impact of productivity or mark-up disturbances and to study alternative monetary and fiscal policies. The impact of the fiscal multipliers on output and the output gap can be quantified showing that a short-run increase in public spending has a multiplier less than one on output and a much smaller multiplier on the output gap, while a decrease in short-run taxes has a positive multiplier on output, but negative on the output gap. In the AS-AD graphical view, optimal policy simplifies to nothing more than an additional line, IT, along which the trade-off between the objective of price stability and that of stabilizing the output gap can be optimally exploited. The framework is also suitable for studying a liquidity-trap environment and possible solutions.
1 Introduction

This work presents a simple New-Keynesian model illustrated by Aggregate Demand (AD) and Aggregate Supply (AS) graphical analysis. In its simplicity, the framework features most of the main characteristics emphasized in the recent literature. The AD and AS equations are derived from an intertemporal model of optimizing behavior by households and firms respectively.

The AD equation is derived from households' decisions on intertemporal consumption allocation. A standard Euler equation links consumption growth to the real interest rate, implying a negative correlation between prices and consumption. A rise in the current price level increases the real interest rate and induces consumers to postpone consumption. Current consumption falls.

The AS equation derives from the pricing decisions of optimizing firms. In the long run, prices are totally flexible and output depends only on real structural factors. The equation is vertical. In the short run, however, a fraction of firms keep prices fixed at a predetermined level, implying a positive relationship between other firms' prices, which are not constrained, and marginal costs, proxied by the output gap. The AS equation is a positively sloped price-output function.

As in Keynesian theory, the model posits some degree of short-run nominal rigidity. Nominal rigidity can be explained by the fact that price setters have some monopoly power, so that they incur only second-order costs when they do not change their prices. In the long run, the model maintains the classical dichotomy between the determination of nominal and real variables, with a vertical AS equation.

The analysis is consistent with the modern central banking practice of targeting short-term nominal interest rates, not money supply aggregates. The mechanism of transmission of interest rate movements to consumption and output stems from the intertemporal behavior of the consumers. By moving the nominal interest rate, monetary policy affects the real interest rate, hence consumption-saving decisions.

This simple framework allows us to analyze the impact of productivity or mark-up disturbances on economic activity and to study alternative monetary and fiscal policies. In particular, we can analyze how monetary policy should respond to various shocks. That is, a microfounded model yields a natural objective function that monetary policy could follow in its stabilization role, namely the utility of consumers. This objective is well approximated by a quadratic loss function in which policymakers are penalized, with certain weights, by deviating from a price-stability target and at the same time by the fluctuations of output around the efficient level. In the AS-AD graphical plot, optimal policy simplifies to just an additional curve (labelled IT for “Inflation Targeting”) along which the trade-off between the two objectives can be optimally exploited.

As Hicks (1937) observed, Keynesian economics is the “economics of the Depression”. The LM is flat because interest rates are too low and any injection of liquidity is absorbed by the public. Monetary policy is ineffective and fiscal policy is the only
possible escape from this “liquidity trap”. A New-Keynesian model gives a new sense to the trap and a new way out. A liquidity trap might arise when the economy is marked by a low nominal interest rate, but in an equilibrium in which the real interest rate is too high. Consumers have a strong incentive to save and postpone consumption. The economy is in a slump. But although monetary policy has lost its standard instrument, it can still lower the real interest rate to stimulate consumption by creating inflationary expectations. This new view on the liquidity trap and the escape can be represented within the same AS-AD graphical framework.

Moreover, it is possible to quantify the effect of the fiscal multipliers on output and on the output gap. An increase in short-run public spending improves output with a multiplier of less than one and the output gap with a much lower multiplier, in the range of 0.1-0.2. A short-run reduction in taxes improves output but worsens the output gap; moreover, short-run fiscal policy changes in either expenditure or taxes do not affect the output gap if they become permanent. However, a reduction in long-term taxes or spending can benefit short-run output and the output gap and still be consistent with fiscal sustainability.

The structure of the article is the following. Section 2 discusses the literature. Section 3 derives the AD equation and Section 4 the AS equation. Section 5 presents the AS-AD model and its graphical representation. Readers not interested in technicalities can skip Sections 3 and 4 or just grasp a few highlights from them and concentrate on Section 5 onward. Sections 6 and 7 analyze the way the equilibrium changes when there are productivity shocks and mark-up shocks. Section 8 studies the fiscal multiplier, and Section 9 analyzes the liquidity-trap solutions. Section 10 sets out a graphical interpretation of the optimal monetary policy.

2 Background literature

This small essay stands on the giants’ shoulders of a literature that has developed since the 1980s. A comprehensive review, naturally, is well beyond our scope here. A brief survey must include the menu cost models of Mankiw (1985), the monopolistic-competition model of Blanchard and Kyotaki (1987) and the dynamic models of Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1997) and Clarida et al. (1999) and culminate with the comprehensive treatments of Woodford (2003) and Galí (2008). For the general readership of this paper, technicalities are kept to the lowest possible level; for a thorough analysis, interested readers are referred to the various chapters of Woodford. The section on optimal policy draws on the work of Giannoni and Woodford (2002), also borrowing ideas and terminology from Svensson (2007a,b). The reference point for the liquidity-trap solution using unconventional monetary policy is Krugman (1998).

There are some obvious limitations to the analysis, essentially the price paid for the simplifications needed for a graphical analysis (chiefly the assumption of a two-period economy). It follows that the short-run AS equation resembles the New-
Classical Phillips familiar to undergraduates more than a New-Keynesian Phillips curve with forward-looking components. The model cannot properly analyze inflation dynamics, disinflation and related questions. The AD equation can dispense with interest-rate rules, which might be an asset or a liability, since long-term prices are anchored by monetary policy and current movements in the interest rate are sufficient to determine equilibrium with no need of feedback effects on economic activity. The dynamic aspects of the stabilizing role of monetary policy are missing, but not their qualitative results.

One theory beats another by demonstrating superiority in practical application. This essay is motivated in part by the fact that New-Keynesian economics has not changed the small models used for undergraduate courses. Indeed, variations on the Hicksian view of the Keynes's *General Theory* are still present in the most widely used textbooks. Most of the critiques of current models come from within. Mankiw (2006) argues that New Keynesian research has little impact on practical macroeconomics. This view is even more forceful given that the recent policy debate on the efficacy of fiscal policy stimulus has mostly been couched in terms of Keynesian multipliers. “New Classical and New Keynesian research has had little impact on practical macroeconomists who are charged with the messy task of conducting actual monetary and fiscal policy. It has also had little impact on what teachers tell future voters about macroeconomic policy when they enter the undergraduate classroom. From the standpoint of macroeconomic engineering, the work of the past several decades looks like an unfortunate wrong turn.” (Mankiw, 2006, p.21)

Krugman (2000) has argued instead that it would be a shame to excise IS-LM model from the undergraduate curriculum, because current models have not lived up to their promise. “The small models haven’t gotten any better over the past couple of decades; what has happened is that the bigger, more microfounded models have not lived up to their promise. The core of my argument isn’t that simple models are good, it’s that complicated models aren’t all they were supposed to be. (...) It would be a shame if IS-LM and all that vanish from the curriculum because they were thought to be insufficiently rigorous, replaced with models that are indeed more rigorous but not demonstrably better.” (Krugman, 2000, p.42)

Indeed, the large-scale dynamic stochastic general equilibrium DSGE models adopted by many national central banks and international institutions are often too complicated for even the sophisticated reader to grasp the essence of monetary policy making.\footnote{In 2007, Bernanke argued that DSGE models “are unlikely to displace expert judgment”; and after the recent turmoil, this statement might be phrased even more strongly.}

Other works have analyzed simplified versions of New Keynesian models. Romer (2000) presents a modern view of a Keynesian non-microfounded model without the LM equation. The demand side is represented by an aggregate demand-inflation curve derived from the Euler equation and a monetary policy rule. In the model presented here, by contrast, the Euler equation is interpreted as an AD equation.
without an interest-rate rule. Walsh (2002) presents a two-equation model in which the AD equation is derived from the optimal transformation between prices and output desired by an optimizing central bank. This is close to the IT equation used here in the analysis of the optimal monetary policy. Finally, Carlin and Soskice (2005) present a simple three-equation non-microfounded model using a modern approach to central bank operational targets, but it lacks an AD equation and needs two graphs to be displayed.\(^2\)

\section{Aggregate Demand}

An important difference between New-Keynesian models and the standard Keynesian IS-LM model is the introduction of optimizing behavior of households and firms. In our simple model, aggregate demand is obtained from households’ decisions on optimal allocation of consumption and aggregate supply from the firm’s optimization problem given households’ labor-supply decisions.

Households get utility from consumption and disutility from work. They optimally choose how to allocate consumption and hours worked across time under a natural resource constraint that bounds the current value of expenditure with the current value of resources available. We develop the analysis in a two-period model, which is enough to characterize intertemporal decisions – another novelty with respect to Keynesian models. The first period represents the short run, the second the long run. In particular, the long run will be such because it displays the classical dichotomy between real and nominal allocations, with monetary policy influencing only long-run prices. In the short run monetary policy is not neutral, owing to the assumption of price rigidity.

Households face an intertemporal utility function of the form

\[ u(C) - v(L) + \beta\{u(\bar{C}) - v(\bar{L})\} \]  

(1)

where \( u(\cdot) \) and \( v(\cdot) \) are non-decreasing functions of consumption, \( C \), and hours worked, \( L \). \( C \) and \( L \) denote consumption and hours worked in the short run, \( \bar{C} \) and \( \bar{L} \) the same variables in the long run. Households derive utility from current and future consumption and disutility from current and future hours worked; \( \beta \) is the factor by which households discount future utility flows, where \( 0 < \beta < 1 \).

Households are subject to an intertemporal budget constraint in which the current value of goods expenditure is constrained by the value of incomes

\[ PC + \frac{\bar{P}\bar{C}}{1+i} = WL + \frac{\bar{W}\bar{L}}{1+i} + T \]  

(2)

\(^2\)Corsetti and Pesenti (2005) and Goodfriend (2004) also present simple analyses, but with less conventional diagrams.
where \( P \) and \( W \) are nominal prices and wages and \( i \) is the nominal interest rate, while \( T \) denotes lump-sum transfers from the government.\(^3\)

Households choose consumption and work hours to maximize utility (1) under the flow budget constraint (2). In each period the marginal rate of substitution between labor and consumption is equated to the real wage

\[
\frac{v_l(L)}{u_c(C)} = \frac{W}{P},
\]

\[
\frac{v_l(\bar{L})}{u_c(C)} = \frac{\bar{W}}{\bar{P}},
\]

where \( u_c(\cdot) \) and \( v_l(\cdot) \) denote the marginal utility of consumption and the marginal disutility of hours worked, respectively. The intertemporal dimension of the optimization problem is captured by the Euler equation, which characterizes how households allocate consumption across the two periods

\[
\frac{u_c(C)}{\beta u_c(C)} = (1 + i) \frac{P}{\bar{P}} = 1 + r. \tag{3}
\]

The optimality condition (3) describes the equilibrium relationship between the intertemporal marginal rate of substitution in consumption and the intertemporal relative price of consumption, given by the real interest rate, \( r \). We can obtain further insights into equation (3) by assuming isoelastic utility of the form \( u(C) = C^{1-\tilde{\sigma}^{-1}}/(1 - \tilde{\sigma}^{-1}) \), in which the marginal utility of consumption is given by \( u_c(C) = C^{-\tilde{\sigma}^{-1}} \) where \( \tilde{\sigma} \) is the intertemporal elasticity of substitution in consumption, with \( \tilde{\sigma} > 0 \). The log of (3) can be simply written as

\[
\bar{c} - c = \tilde{\sigma}r + \tilde{\sigma}\ln \beta \tag{4}
\]

(the lower case letter denotes the log of its respective variable, and I have used the approximation \( \ln(1 + r) \approx r \)).

Equation (4) shows how the real interest rate affects the intertemporal allocation of consumption. Other things being equal, a rise in it induces households to save more and postpone consumption. Consumption growth is positively correlated with the real interest rate. Low real interest rates are a disincentive for saving and fuel current consumption. We can also write (4) as

\[
\bar{c} - c = \tilde{\sigma}[i - (\bar{p} - p)] + \tilde{\sigma} \ln \beta \tag{5}
\]

in which the real rate is expressed explicitly as the nominal interest rate less inflation, between the long- and the short-run. This Euler equation (5) will be interpreted as an aggregate-demand equation, in that it marks the negative relationship between

\(^3\)Firms’ profits are also included in \( T \), for simplicity.
current consumption, \( c \), and prices, \( p \). When current prices rise so does the real interest rate, which implies higher savings and lower current consumption.

We can elaborate further on the previous equation by noticing that in each period equilibrium output is equal to consumption plus public expenditure

\[
Y = C + G, \\
\bar{Y} = \bar{C} + \bar{G},
\]

where \( Y \) denotes output and \( G \) is public expenditure. These equations imply in a first-order approximation that

\[
y = s_c c + g,
\]

and

\[
\bar{y} = s_c \bar{c} + \bar{g},
\]

where \( s_c \) denotes the steady-state share of consumption in output and \( g \) that of public spending. Substituting these equations into (5), we obtain

\[
y = \bar{y} + (g - \bar{g}) - \sigma (i - (\bar{p} - p)) - \sigma \ln \beta
\]

which thus becomes a negative relation between current output and current prices, where \( \sigma = \tilde{\sigma} s_c \).

Before moving to the AS equation, let us further investigate the implications of enriching the model by considering taxes both on consumption and on wages. The intertemporal budget constraint becomes

\[
(1 + \tau_c)PC + \frac{(1 + \bar{\tau}_c)\bar{P}\bar{C}}{1 + \bar{i}} = (1 - \tau_l)WL + \frac{(1 - \bar{\tau}_l)\bar{W}\bar{L}}{1 + \bar{i}} + T
\]

where \( \tau_c \) denotes the tax rate on consumption expenditure, \( \tau_l \) that on labor income. The optimality condition between consumption and labor is now

\[
\frac{v_l(L)}{u_c(C)} = \frac{(1 - \tau_l) W}{(1 + \tau_c) P}.
\]

Similarly for the long run. The Euler equation too changes to imply a log-linear AD equation of the form

\[
y = \bar{y} + (g - \bar{g}) - \sigma [i - (\bar{p} - p) - (\bar{\tau}_c - \tau_c)] - \sigma \ln \beta,
\]

which is also shifted by movements in consumption taxes.
4 Aggregate Supply

The aggregate supply side of the model is derived from firms’ pricing decisions, given labor supply. There are many producers offering goods differentiated according to consumers’ tastes. Producers have some monopoly power in pricing, as the market is one of monopolistic competition. That is, firms have some leverage on their price but are small with respect to the overall market. The generic producer $j$ faces demand

$$ Y(j) = \left( \frac{P(j)}{P} \right)^{-\theta} (C + G) \quad (9) $$

where $\theta$, with $\theta > 0$, is the elasticity of substitution of consumer preferences among goods and $P(j)$ is the price of the variety $j$ produced by firm $j$.\footnote{The demand equation (9) can be obtained in a rigorous way by positing that $C$ is a Dixit-Stiglitz aggregator of all the goods produced in the economy. See Dixit and Stiglitz (1977) and Woodford (2003, ch. 3).} The only factor of production is labor, which is utilized by a linear technology $Y(j) = AL$ where $A$ is a productivity shock. In the short run, the profits of the firm $j$ are given by

$$ \Pi(j) = (1 - \tau_y)P(j)Y(j) - (1 + \tau_w)WL, \quad (10) $$

where $\tau_y$ is the tax rate on sales and $\tau_w$ that on labor costs.

Equations (9) and (10) apply both to the short and to the long run. However, in the short run, a fraction $\alpha$ (with $0 < \alpha < 1$) of firms are assumed to maintain their prices fixed at the predetermined level $P^e$. At this level, firms adapt production to demand (9). The remaining fraction $1 - \alpha$ of firms maximize profits (10). But in the long run all firms can adjust their prices in an optimal way.

The assumption of sticky prices, a classic in the Keynesian tradition, has its rationale in a New-Keynesian model because of monopolistic competition. Firms make positive profits, and losses from not changing prices are of second-order importance compared to unmodelled menu costs (see Mankiw, 1985).

4.1 The short run

Under monopolistic competition the firms can influence demand (9) by choosing the prices of their goods, but each one is too small to influence the aggregate price level $P$ or aggregate consumption $C$. Prices $P(j)$ are set to maximize (10) given demand (9). At the optimum, pricing is a markup over marginal costs

$$ P(j) = (1 + \tilde{\mu}) \frac{W}{A}, \quad (11) $$

where the mark-up $\tilde{\mu}$ is defined as

$$ \tilde{\mu} \equiv \frac{\theta}{\theta - 1} \frac{1 + \tau_w}{(1 - \tau_y)} - 1. $$
We can write equation (11) as
\[
\frac{P(j)}{P} = (1 + \hat{\mu}) \frac{W}{PA},
\]
and substitute in real wages using equation (7) to obtain
\[
\frac{P(j)}{P} = \frac{(1 + \mu)}{A} \frac{v_t(L)}{u_c(C)} = \frac{(1 + \mu)}{A} \frac{L^n}{C^{-\sigma - 1}}. \tag{12}
\]

In equation (12), \(\mu\) represents an aggregate measure of the mark-up in the economy, given by a combination of the mark-up, defined as \(\mu_a \equiv \theta/(\theta - 1)\), and the tax rates, as:
\[
\mu \equiv \mu_a \frac{(1 + \tau_w)}{(1 - \tau_y)} \frac{(1 + \tau_c)}{(1 - \tau_l)} - 1. \tag{13}
\]

In (12), an isoelastic disutility of labor of the form \(v(L) = L^{1+\eta}/(1 + \eta)\) is also assumed.

At this point, it is important to define the natural level of output, \(Y_n\), as the level that would obtain in a model with flexible prices and a marginal rate of substitution between labor and consumption proportional to the real wage
\[
\frac{v_t(Y_n/A)}{u_c(Y_n - G)} = \frac{(1 - \tau_l)}{(1 + \tau_c)} \frac{W}{P},
\]

implying
\[
\frac{v_t(Y_n/A)}{u_c(Y_n - G)} = \frac{A}{(1 + \mu)}, \tag{14}
\]

where we have also used the resource constraint \(Y = C + G\) and the production function \(Y = AL\). In a log-linear approximation, we get
\[
y_n = \frac{1 + \eta}{\sigma^{-1} + \eta} a + \frac{\sigma^{-1}}{\sigma^{-1} + \eta} g - \frac{1}{\sigma^{-1} + \eta} \mu. \tag{15}
\]

The natural level of output rises with productivity and public expenditure and falls when the aggregate mark-up increases because of an increase in taxes or in monopoly power.

Now let us turn to equation (12), which can be written using the definition of the natural rate of output, the resource constraint and technology, as
\[
\frac{\tilde{P}}{P} = \left( \frac{Y}{Y_n} \right)^{\eta} \left( \frac{Y - G}{Y_n - G} \right)^{\tilde{\sigma} - 1}
\]
in which it is assumed that \(P(j) = \tilde{P}\) for all the firms that can adjust their prices. In a log-linear approximation, we can write
\[
\tilde{p} - p = (\sigma^{-1} + \eta)(y - y_n). \tag{16}
\]
Noticing that the general price level is a weighted average of the fixed and the flexible prices
\[ p = \alpha p^e + (1 - \alpha) \tilde{p} \]
(16) can be written as
\[ p - p^e = \kappa (y - y_n), \]
a short-run aggregate-supply equation relating unexpected movements in prices with the output gap (the difference between the actual and natural level of output). When output exceeds the natural rate level – a positive output gap – there is upward pressure on prices, driving them above their predetermined levels. The parameter \( \kappa \equiv (1 - \alpha)(\sigma^{-1} + \eta)/\alpha \) measures the slope of the AS curve. The AS will be flatter – movements in the output gap create less variation in prices – when the fraction of sticky-price firms is larger. On the other side, it will be steeper when there are more flexible-price firms. The natural level of output, \( y_n \), is reached when all firms have flexible prices or, in a sticky-price environment, when actual prices are equal to expected prices, \( p = p^e \).

The AS equation (17) is not really a novel feature of New-Keynesian models. It is known in undergraduate textbooks as New-Classical Phillips curve. Phelps (1967) and Lucas (1971) have derived a similar equation on different principles based on imperfect information for firms’ decisions.\(^5\)

### 4.2 The long run

In the long run, all firms can freely set their prices. Output and consumption will find their natural level, namely
\[
\begin{align*}
\bar{y}_n &= \frac{1 + \eta}{\sigma^{-1} + \eta} \bar{a} + \frac{\sigma^{-1}}{\sigma^{-1} + \eta} \bar{g} - \frac{1}{\sigma^{-1} + \eta} \bar{\mu}, \\
\bar{c}_n &= \frac{1 + \eta}{s_c(\sigma^{-1} + \eta)} \bar{a} - \frac{\eta}{s_c(\sigma^{-1} + \eta)} \bar{g} - \frac{1}{s_c(\sigma^{-1} + \eta)} \bar{\mu}.
\end{align*}
\]
The long-run Phillips curve is vertical and the model displays the classical dichotomy between nominal and real variables. Monetary policy is neutral with respect to the determination of real variables, but fiscal policy is not. A rise in tax rates, in any form, increases the mark-up and reduces both output and consumption. More public spending increases output but in general reduces consumption. When the disutility of labor is linear, i.e. \( \eta = 0 \), output rises one-to-one with public expenditure, while consumption remains unchanged. Greater long-run productivity growth will raise the natural level of both output and consumption.

\(^5\)See Ball et al. (1988) for a New Keynesian interpretation. The most recent New Keynesian literature has focused on forward-looking and staggered price-setting behavior following Calvo (1983) to derive a New Keynesian Phillips curve that incorporates future expectations. This complication goes beyond our present pedagogical scope and in any case would not affect the qualitative results of the following sections. Others have assumed staggered pricing using overlapping contracts as in Taylor (1979) or the costly-adjustment model of Rotemberg (1982).
4.3 Policies

A number of different monetary and fiscal policy instruments are available and the model should be closed to determine all the relevant macro variables by specifying a path for those instruments. In the long run, we assume that monetary policy controls and determines \( \bar{\rho} \) in line with the result of long-run neutrality. In the short run, we assume that it controls the nominal interest rate, \( i \). Fiscal policy instead sets the path of taxes for the short and the long run, \( \{ \tau_l, \tau_w, \tau_c, \tau_y, \tilde{\tau}_l, \tilde{\tau}_w, \tilde{\tau}_c, \tilde{\tau}_y \} \) and public expenditure, for the short and the long run \( \{ g, \bar{g} \} \), together with the transfer \( T \) to balance the intertemporal budget constraint

\[
\tau_y PY + \tau_c PC + (\tau_w + \tau_l) WL + \frac{\tau_y \tilde{P}Y + \tau_c \tilde{P}C + (\tilde{\tau}_w + \tilde{\tau}_l) \tilde{W} \tilde{L}}{1 + i} = PG + \bar{P}\bar{G} + T. \tag{18}
\]

Given these policies, in the short run prices and output are determined by the AD equation (8) and the AS equation (17). Movements in the monetary policy instrument \( i \) are now not neutral with respect to output because of price rigidity.

4.4 The efficient level of output

The “efficient” level of output is that which maximizes the utility of consumers under the resource constraints of the economy. It is the level that maximizes

\[
u(C) - v(L)
\]

under the resource constraint

\[Y = C + G,
\]

and technology

\[Y = AL.
\]

The optimality condition defines the efficient level of output, \( Y_e \), as a function of technology and public spending:

\[
\frac{u_c(Y_e - G)}{v_l(Y_e/A)} = \frac{1}{A}.
\]

In log-linear form, it reads

\[
y_e = \frac{1 + \eta}{\sigma^{-1} + \eta} a + \frac{\sigma^{-1}}{\sigma^{-1} + \eta} g. \tag{19}
\]

A comparison of (19) with (15) shows that productivity and public spending shift the two definitions of output in equal proportion. Mark-up shocks, however, are a source of inefficiencies, as they shift the natural but not the efficient level of output. As is shown later, the efficient level of output is the welfare-relevant measure under certain conditions, i.e. when \( y_e \) and \( y_n \) are close, requiring a small \( \mu \).
Figure 1: The AS equation is a positive relationship between prices and output. Higher output increases real wages and firms’ real marginal costs. The firms that can adjust their prices react by increasing them. AS crosses through the point \((p^e, y_n)\).

5 The AS-AD model

Now our short-run equilibrium can be interpreted through graphical analysis with the AS and AD equations. The AS equation is given by a sort of New Classical Phillips curve

\[ p - p^e = \kappa(y - y^n), \]  

which shows a positive relationship between prices and output. An increase in output leads to higher real marginal costs. Firms can protect profit margins by price hikes.\(^6\)

Figure 1 plots the AS curve with prices on the vertical and output on the horizontal axis. There is a positive relationship between prices and output. The slope is \(\kappa\). Since \(\kappa\) depends on the fraction of sticky-price firms, the larger this fraction, lower the slope.

\(^6\)Sticky-price firms will meet the higher demand by increasing production.
Figure 2: The AS curve shifts to the right when the short-run current natural rate of output increases ($y_n \uparrow$) because of an increase in short-run productivity ($a \uparrow$), an increase in short-run public spending ($g \uparrow$), or a fall in short-run mark-up ($\mu \downarrow$), itself due to either a fall in short-run monopoly power ($\mu_\theta \downarrow$) or in short-run tax rates ($\tau_c \downarrow, \tau_w \downarrow, \tau_t \downarrow, \tau_y \downarrow$).

and the flatter the AS equation. In this case, movements in real economic activity produce small changes in the general price level. A useful observation in plotting the AS equation and understanding its movements is that it crosses the point ($p^e, y_n$). In particular, we have seen that the natural level of output depends on public expenditure, productivity and the mark-up. An increase in productivity or in public expenditure or a reduction in the mark-up through lower taxes all raise the natural level of output. In these cases, Figure 2 shows that the vertical line corresponding to $y_n$ shifts rightward to the new level $y'_n$. The AS equation shifts to the right and crosses the new pair ($p^e, y'_n$). Vice versa in the case of a fall in the natural level of output.

In the traditional AS-AD textbook model, the AD equation represents the combination of prices and output such that goods and financial markets are in equilibrium. The cornerstone of the building block behind the AD equation is the IS-LM model.
In that framework, the AD curve is negatively sloped, because a rise in prices increases the demand for money. For a given money supply, interest rates should rise to discourage the increased desire for liquidity. Investment falls along with demand and output.

The AD equation of our New-Keynesian model originates from different principles. First, there is no LM equation, since the instrument of monetary policy is the nominal interest rate and not money supply. Second, the model posits intertemporal and optimizing decisions by households, which show up directly in the specification of the AD equation. As the previous section has shown, the New Keynesian AD equation is

$$y = \bar{y} + (g - \bar{g}) - \sigma [i - (\bar{p} - p) - (\bar{\tau}_c - \tau_c)] - \sigma \ln \beta,$$

which is a negative relationship between prices and output, with slope of $-1/\sigma$. When prices rise, for a given path of the other variables and in particular of the nominal interest rate, the real interest rate rises. This prompts households to increase saving and postpone consumption. Current consumption falls along with current production. When $\sigma$ is high, small movements in prices and in the real rate produce larger movements in savings and a larger fall in consumption and output: the AD equation becomes flatter; and conversely when $\sigma$ is low. To simplify the AD graph, we assume to start with that $\bar{p} = p^e$ and that the curve also crosses the natural level of output. This assumption requires setting the nominal and real interest rate at the natural level – the level that would occur under flexible prices. Figure 3 plots the AD equation under these assumptions.

Several factors can move AD. First, monetary policy can shift the curve by affecting the nominal interest rate. A lower nominal interest rate ($i \downarrow$) shifts the curve to the right, since it lowers the real interest rate proportionally. Consumers reduce saving and step up current consumption, given prices (see Figure 4). Short-run movements in fiscal policy can also move the AD equation by altering public expenditure and consumption taxes. In particular, an increase in short-run public spending ($g \uparrow$) shifts the equation to the right because, for given prices, it increases current output. A similar movement follows a lowering of short-run consumption taxes ($\tau_c \downarrow$). In this case, the current relative to the future cost of buying goods is diminished inducing higher consumption. The long-run monetary and fiscal policy stances affect the short-run movements in the equation. An increase in long-run prices ($\bar{p} \uparrow$) lowers the real interest rate and drives current consumption and output up, shifting the curve to the right. Long-run fiscal policy works through its impact on the long-run natural level of consumption, since $\bar{c}_n = \bar{y}_n - \bar{g}$. A reduction in future payroll and sales taxes, ($\bar{\tau}_y \downarrow, \bar{\tau}_w \downarrow, \bar{\tau}_l \downarrow$), reduces the long-run mark-up and so increases the natural level of consumption, shifting AD upward. A decrease in consumption taxes ($\tau_c \downarrow$) has a similar effect on the mark-up, but a different impact on current consumption because of consumers’ desire to postpone consumption so as to exploit the lower future taxation. This second channel dominates. Changes in long-run productivity can also cause an increase in the long-run natural level of consumption and an upward shift.
Figure 3: AD is a negative relationship between prices and output. As current prices increase, the real interest rate rises and consumers save more. Current consumption falls along with production.
Figure 4: The AD curve shifts upward when the short-run nominal interest rate falls ($i \downarrow$), short-run consumption taxes fall ($\tau_c \downarrow$), short-run public spending increases ($g \uparrow$), long-run prices increase ($\bar{p} \uparrow$), or the future natural level of consumption rises ($\bar{c}_n \uparrow$), due to an increase in long-run productivity ($\bar{a} \uparrow$), a reduction in long-run public spending ($\bar{g} \downarrow$), a fall in long-run monopoly power ($\bar{\mu}_g \downarrow$), a fall in long-run payroll and income taxes ($\bar{\tau}_y \downarrow$, $\bar{\tau}_w \downarrow$, $\bar{\tau}_l \downarrow$), or an increase in long-run consumption taxes ($\bar{\tau}_c \uparrow$).
6 Productivity shocks

We start some comparative static exercises, assuming that in the initial equilibrium AS and AD cross at point $E$, as shown in Figure 5, in which $p = p^e = \bar{p}$ and the economy is at the natural level of output. It is also assumed that the natural and efficient levels of output initially coincide, $y_n = y_e$. The way the equilibrium changes in response to different shocks and how monetary policy should react to restore stability in prices and output gap, if possible, are then analyzed. Let us begin with the analysis of productivity shocks.
6.1 A temporary productivity shock

First, we analyze the case in which the economy undergoes a temporary productivity gain, meaning that productivity rises in the short run but does not vary in the long run. Starting from the equilibrium $E$, shown in Figure 6, the short-run natural level of output rises to $y_n'$ and $AS$ shifts downward, crossing $E''$, as discussed in the previous section. The $AD$ equation is not affected by movements in current productivity, and the new equilibrium is found at the intersection of the new $AS$ equation $AS'$, with the old $AD$ equation.

The adjustment from equilibrium $E$ to $E'$ occurs as follows. A temporary increase in productivity lowers real marginal costs. The firms that can adjust their prices lower them. The real interest rate falls, stimulating consumption. Output increases, but not enough to match the rise in the natural level, because of sticky prices. The economy reaches the equilibrium point $E'$ with lower prices and higher output, but a negative output gap. The efficient level of output rises in the same proportion as the natural level.

This is the new equilibrium without any policy intervention. An interesting ques-
tion is how monetary policy should respond to the shocks in order to close the output gap and/or stabilize prices, if both objectives can be reached simultaneously. In this case, monetary policy can attain both by bringing the economy to equilibrium $E''$. By lowering the nominal interest rate, monetary policy shifts AD upward to the point at which it crosses $E''$. The curve thus shifts to $AD''$. The rise in output also increases firms’ marginal costs, driving prices up. Production expands. In $E''$ prices are stable at the initial level. An accommodating monetary policy can thus achieve both the natural level of output and stable prices. As will be discussed later, this is also the optimal policy, maximizing the utility of the consumers.

6.2 A permanent productivity shock

Figure 7 analyzes the case of a permanent productivity shock. As in the previous case, the short-run natural and efficient level of output increases and the $AS$ curve shifts downward through $E'$. However, now the $AD$ equation shifts upward, because the future natural level of consumption rises along with long-run productivity. Households want to increase current consumption because they want to smooth the future increase. The $AD$ equation shifts exactly to intersect $E'$. And this is the new equilibrium.

In the initial equilibrium, the nominal interest rate was set equal to the natural real rate of interest

$$i = r_n = \sigma^{-1}(\bar{y}_{n} - y_{n}) + \sigma^{-1}(\bar{\gamma} - \gamma) + (\bar{\tau}_{c} - \tau_{c}) - \ln \beta.$$  

A permanent gain in productivity does not change the natural real rate of interest, because both $\bar{y}_{n}$ and $y_{n}$ increase in the same proportion: this is why the new equilibrium requires no change in monetary policy to achieve price stability while closing simultaneously the output gap. Later, I will show that this equilibrium outcome corresponds to the optimal monetary policy.

6.3 Optimism or pessimism on future productivity

Consider now the case of an expected increase in long-run productivity. This can be taken either as an increase that will really occur, or just an optimistic belief about future output growth or a combination of the two. Conversely, a decrease in long-run productivity can also be interpreted as a pessimistic belief on future growth. The case of optimism is analyzed in Figure 8.

$AS$ does not move, since there is no change in current productivity. $AD$ does shift upward, because households expect higher consumption in the future and for the smoothing motive they want to increase their consumption immediately. Output expands driving up real wages and real marginal costs, so that firms adjust prices upward. The economy reaches the equilibrium point $E'$ with higher prices and production higher than the natural and efficient, level. What should monetary policy do
Figure 7: A permanent productivity shock: $a \uparrow$ and $\bar{a} \uparrow$. AS shifts to $AS'$ and $AD$ to $AD'$. The equilibrium moves from $E$ to $E'$ without any monetary policy intervention.
Figure 8: An increase in long-run productivity: $\bar{a} \uparrow$. $AD$ shifts up to $AD'$. The equilibrium moves from $E$ to $E'$. An increase in the nominal interest rate can bring the equilibrium back to the starting point.
to stabilize prices and close the output gap? It should counter future developments in productivity or such optimistic beliefs by raising the nominal interest rate so as to bring $AD$ back to the initial point $E$.

Some lessons can be drawn from these analyses. Regardless of the properties of the shock—temporary, permanent or expected—monetary policy can always move interest rates to stabilize prices and the output gap simultaneously. But the direction of the movement depends on the nature of the shock. When shocks are transitory, monetary policy should be expansionary; when permanent, it should be neutral; and with merely expected productivity shocks, it should be restrictive.

7 Mark-up shocks

When there are productivity shocks, monetary policy does not face a trade-off between stabilizing prices and offsetting the output gap. With mark-up shocks, things are different. First, the efficient level of output does not move, while the natural level does. Second, there is a trade-off between stabilizing prices and reaching the efficient level of output.

This section considers only temporary mark-up shocks, leaving other analyses to the reader. Among such shocks, we concentrate on those that move the $AS$ equation. The shock envisaged here is a short-run increase in the mark-up ($\mu \uparrow$), due to an increase in monopoly power ($\mu_0 \uparrow$) or a rise in tax rates ($\tau_w \uparrow$, $\tau_l \uparrow$, $\tau_y \uparrow$). Another appealing interpretation of such a mark-up shock is as of a variation in the price of commodities that are inelastically demanded as factors of production. An important example is oil.

Consider then a temporary increase in the mark-up (see Figure 9). The $AS$ curve shifts upward following the fall in the natural level of output, but the efficient level of output is unchanged. Firms raise prices because of the higher mark-up. The real interest rate goes up and households increase savings and postpone consumption. Demand falls along with output. The economy reaches equilibrium $E'$ with a contraction in output and higher prices: a situation dubbed “stagflation”. Now, monetary policy does face a dilemma: between maintaining stable prices and keeping the economy at the efficient level of production. To attain the price objective the nominal interest rate should be raised so as to further increase the real rate and damp down economic activity. In this case, $AD$ shifts downward to $AD'$ and the economy reaches equilibrium $E''$. To obtain the output objective, policymakers should cut the nominal interest rate to stimulate economic activity and consumption. In this case, $AD$ shifts to $AD''$ and the economy reaches equilibrium $E'''$. 
Figure 9: A temporary increase in the mark-up: $\mu_\theta \uparrow$. The natural level of output $y_n$ falls while the efficient level of output $y_e$ does not move. $AS$ shifts to $AS'$. The equilibrium moves from $E$ to $E'$. By raising the nominal interest rate, monetary policy can stabilize prices and reach equilibrium $E''$. By lowering it, the efficient level of output can be achieved at equilibrium point $E'''$. There is a trade-off between the two objectives, stabilizing prices and reaching the efficient level of output.
8 Fiscal multipliers

So far, we have conducted exercises in comparative statics through perturbations originating from productivity or mark-up shocks, to examine how monetary policymakers should react in order to stabilize prices or close the output gap. In this section, the focus is instead on fiscal policy and in particular on the impact of alternative fiscal policy stances on output and the output gap.

To shed light on the fiscal multipliers, let us consider only the components of the equation (21) that are influenced by fiscal policy

\[ y = \bar{c}_n + g - \sigma[p - (\bar{\tau}_c - \tau_c)], \]

in which we can substitute (20) for \( p \) to obtain

\[ y = \left[ g + \bar{c}_n + \kappa \sigma y_n + \sigma(\bar{\tau}_c - \tau_c) \right] \frac{1}{1 + \kappa \sigma}, \]

again disregarding the terms unaffected by fiscal policy. Recalling the definition of the natural level of consumption and output, we can write

\[ y = m_g g - m_g \bar{g} - m_\tau \tau - m_\tau \bar{\tau} - m_{\tau c} \tau_c + m_{\tau c} \bar{\tau}_c, \quad (22) \]

where the parameters are all defined in terms of the primitives of the model, as shown in Table 1. Moreover, all the tax rates except for consumption taxes are collapsed in \( \tau \), so that \( \tau = \tau_l + \tau_w + \tau_y \); similarly for \( \bar{\tau} \). I call the coefficients in (22) “multipliers”, but actually most if not all do not have multiplying effects on output in the Keynesian sense. In particular, an increase in current public expenditure, \( g \uparrow \), increases output. The increase is one-to-one in the special case of linear disutility of labor, i.e. \( \eta = 0 \); otherwise the increase is less than proportional.

Greater short-run public expenditure \( (g \uparrow) \) increases output because it stimulates aggregate demand. But, prices rise as demand increases, producing an increase in the real interest rate and a decrease in consumption. There is a crowding out effect of public spending on private spending, except when the disutility of labor is linear, \( \eta = 0 \).

By contrast, higher long-run public expenditure, \( \bar{g} \uparrow \), decreases current output, because it depresses the long-run natural level of consumption. Anticipating this, households reduce current consumption, and production contracts. Higher short-run and long-run sales and payroll taxes have also depressing effects on current output, but through different channels. An increase in short-run taxes, \( \tau \uparrow \), moves the AS curve upward and pushes up prices along AD, raising the real interest rate and reducing current consumption and thus output.

An increase in long-run taxes, \( \bar{\tau} \uparrow \), reduces the long-run natural level of consumption and thus affects current consumption. This produces a downward shift in AD.
An increase in short-run consumption taxes, $\tau_c \uparrow$, reduces current output through two channels. First, the increase acts as a mark-up shock, driving up prices and shifting the AS curve upward. Second, the relative price of current consumption rises with respect to future consumption, so the AD curve shifts as savings increase and current consumption falls. An increase in future consumption taxes, $\bar{\tau}_c \uparrow$, also affects the long-run natural level of consumption adversely, but it reduces the relative price of current against future consumption. Both channels now operate through the AD equation, and the latter dominates, implying that a rise in long-run consumption taxes increases current output. With linear disutility, $\eta = 0$, the two channels offset one another.

### Table 1: Fiscal multipliers

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_g$</td>
<td>$\frac{1}{(1+\kappa\sigma)} + \frac{\kappa}{(\sigma^{-1}+\eta)(1+\kappa\sigma)}$</td>
</tr>
<tr>
<td>$m_{\bar{g}}$</td>
<td>$\frac{\eta}{(\sigma^{-1}+\eta)(1+\kappa\sigma)}$</td>
</tr>
<tr>
<td>$m_\tau$</td>
<td>$\frac{\kappa\sigma}{(\sigma^{-1}+\eta)(1+\kappa\sigma)}$</td>
</tr>
<tr>
<td>$m_{\bar{\tau}}$</td>
<td>$\frac{1}{(\sigma^{-1}+\eta)(1+\kappa\sigma)}$</td>
</tr>
<tr>
<td>$m_{\tau_c}$</td>
<td>$\frac{\kappa\sigma}{(\sigma^{-1}+\eta)(1+\kappa\sigma)} + \frac{\sigma}{(1+\kappa\sigma)}$</td>
</tr>
<tr>
<td>$m_{\bar{\tau}_c}$</td>
<td>$m_{\bar{g}}$</td>
</tr>
</tbody>
</table>

Whether these parameters are multipliers or not depends on the value of the primitive parameters. To pin these values down, let us use some calibrations from the literature. In our model, $\alpha$ measures the fraction of price setters who have fixed prices. In Calvo’s model, this is related to the duration of prices $D$, given by $D = 1/(1 - \alpha)$, which is usually assumed to be three quarters for the United States. We can loosely calibrate $\alpha = 0.66$. I also experiment with greater price rigidity, setting $\alpha = 0.75$. The intertemporal elasticity of substitution is usually assumed between one half and one. I set $\sigma = 0.5$ and experiment for a unitary elasticity of substitution $\sigma = 1$. The parameter $\eta$ can be interpreted as the inverse of the Frisch elasticity of labor supply. Values around 5 are reasonable in micro studies while estimates of DSGE models point to 1, as in Smets and Wouters (2003). I set $\eta$ equal to 0.2 and experiment for 1.

Table 2 evaluates the multipliers of (22) using the several combinations of the above parameters. Interestingly, all the multipliers are less than one, and each fiscal instrument has a less than proportional impact on output. In particular a 1% increase in public expenditure with respect to GDP, in the short run, increases output by
between 0.75% and 0.96%, depending on the calibration of the parameters. A short-run increase in taxes reduces output by a factor ranging from 0.1 to 0.3 while the impact of an increase in the consumption taxes, in the short run, depends greatly on the value taken by the intertemporal elasticity of substitution. With a low value, this factor ranges around 0.5, while with a unitary elasticity of substitution it rises to 0.95. The factors of proportionality of long-run taxes and public spending are smaller, ranging from 0.05 to 0.30 depending on the calibration. In some calibrations, the multiplier on long-run taxes is as high as 0.60.

<table>
<thead>
<tr>
<th>$\alpha = 0.66$, $\sigma = 0.5$, $\eta = 0.2$</th>
<th>$m_g$</th>
<th>$m_{\bar{g}}$</th>
<th>$m_\tau$</th>
<th>$m_{\bar{\tau}}$</th>
<th>$m_{\tau c}$</th>
<th>$m_{\bar{\tau} c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.75$, $\sigma = 0.5$, $\eta = 0.2$</td>
<td>$0.96$</td>
<td>$0.06$</td>
<td>$0.16$</td>
<td>$0.29$</td>
<td>$0.48$</td>
<td>$0.05$</td>
</tr>
<tr>
<td>$\alpha = 0.66$, $\sigma = 1$, $\eta = 0.2$</td>
<td>$0.94$</td>
<td>$0.10$</td>
<td>$0.31$</td>
<td>$0.52$</td>
<td>$0.93$</td>
<td>$0.10$</td>
</tr>
<tr>
<td>$\alpha = 0.75$, $\sigma = 1$, $\eta = 0.2$</td>
<td>$0.95$</td>
<td>$0.12$</td>
<td>$0.24$</td>
<td>$0.60$</td>
<td>$0.95$</td>
<td>$0.12$</td>
</tr>
<tr>
<td>$\alpha = 0.66$, $\sigma = 1$, $\eta = 1$</td>
<td>$0.75$</td>
<td>$0.25$</td>
<td>$0.25$</td>
<td>$0.75$</td>
<td>$0.25$</td>
<td>$0.75$</td>
</tr>
<tr>
<td>$\alpha = 0.75$, $\sigma = 1$, $\eta = 1$</td>
<td>$0.80$</td>
<td>$0.30$</td>
<td>$0.20$</td>
<td>$0.30$</td>
<td>$0.80$</td>
<td>$0.30$</td>
</tr>
<tr>
<td>$\alpha = 0.66$, $\sigma = 0.5$, $\eta = 1$</td>
<td>$0.86$</td>
<td>$0.19$</td>
<td>$0.14$</td>
<td>$0.19$</td>
<td>$0.43$</td>
<td>$0.19$</td>
</tr>
<tr>
<td>$\alpha = 0.75$, $\sigma = 0.5$, $\eta = 1$</td>
<td>$0.89$</td>
<td>$0.22$</td>
<td>$0.11$</td>
<td>$0.22$</td>
<td>$0.44$</td>
<td>$0.22$</td>
</tr>
</tbody>
</table>

So far we have considered the impact of fiscal policy on output, showing the conditions under which it is expansionary or not. However, it is not solely in terms of output that a policy can be judged to be expansionary or not. It is surely more appropriate to look at the impact of the multipliers on the output gap and see which policy widens or narrows it the most. Subtracting (15) from (22) and focusing on the terms relevant to fiscal policy, we get

$$y - y_n = m_g(g - \bar{g}) + m_\tau(\tau - \bar{\tau}) - m_{\tau c}(\tau_c - \bar{\tau}_c).$$

(23)

Permanent changes in fiscal policy, whatever the source, do not alter the output gap. Short and long-run movements in any fiscal policy instrument have equal and opposite effects on the gap; the magnitude is given by the long-run multipliers in
An increase in public spending, in the short run, raises output more than the natural level, narrowing the output gap. An increase in sales and payroll taxes, in the short run, reduces output but reduces the natural level still more, thus ultimately narrowing output gap. An increase in consumption taxes also reduces output, but now the contraction is larger than the fall in the natural level, widening the output gap. The effects of long-run fiscal policies on the short-run output gap are exactly opposite; that is, they are of the same sign and magnitude as those on output, since they do not move the current natural level. Table 2 quantifies these multipliers. A short-run—not permanent—increase in public spending narrows the short-run output gap by a factor which ranges between 0.06 and 0.30. But in the more realistic case of a high Frisch elasticity of substitution, \( \eta = 0.2 \), this factor does not exceed 0.12; that is, an increase of 1 percentage point in the ratio of public spending to GDP diminishes the output gap by 0.12%.

We can also analyze the fiscal multipliers and the output gap, computed with respect to the efficient allocation, obtaining

\[
y - y_e = m\bar{g}(g - \bar{g}) - m_{\tau} \tilde{\tau} - m_{\tau_c} \tilde{\tau}_c + m_{\bar{\tau}} \tilde{\bar{\tau}}_c,
\]

which has the same short- and long-run public-spending multipliers as equation (22) and the same taxation multipliers as equation (23). Distorting taxes are inefficient and do not shift the efficient level of output; a short-run spending shock moves the natural and the efficient level of output, proportionally.

Having analyzed the impact of fiscal policy on the output gap, we can now investigate the equilibrium movements using the AS-AD view. Here we consider a short-run increase in public spending, leaving all the other comparative static exercises to the reader. A qualification at this point is that the experiment assumes government can comply with intertemporal solvency constraints by curbing lump-sum transfers as in (18). If lump-sum transfers are not available, one might envisage a scenario in which public spending increases in the short-run while some distorting tax, either in the short or in the long run, is adjusted to offset that increase. A proper analysis requires going beyond the simple model presented here, but we can give a qualitative and loose account of the final results even in our simple model, by studying the combined effects of movements in alternative fiscal instruments.

As shown in Figure 10, an increase in current public spending expands aggregate demand and thus current output, shifting the aggregate demand equation upward from \( AD \) to \( AD' \). At the same time, the natural rate of output increases, together with the efficient level shifting \( AS \) downward. From equation (23), we know that short-run movements in public spending narrow the output gap. Starting from no gap, public spending creates a positive output gap. This means that \( AD' \) and \( AS' \) cross to the right of the new natural level of output, \( y'_n \). Prices rise from the initial equilibrium; the real interest rate rises, reducing consumption as equilibrium \( E \) becomes \( E' \). On the one hand the economy is overheated by a positive output gap, on the other side private consumption is crowded out. To close the output gap and stabilize
Figure 10: A temporary increase in public spending: $g \uparrow$. The natural and efficient level of output $y_n$ increases. AS shifts to $AS'$. The equilibrium moves from $E$ to $E'$ with a positive output gap. By raising the nominal interest rate, monetary policy can stabilize prices and the output gap, reaching equilibrium $E''$. 
prices, policymakers should raise nominal interest rates to increase the real rate, which further dampens private consumption. In this case, \( AD \) moves from \( AD' \) to \( AD'' \), reaching equilibrium \( E'' \).

### 9 Liquidity trap

Hicks (1932) describes the *General Theory* as the “economics of depression,” the reason being that Keynes had said something about an increase in investment not raising the nominal interest rate. In the traditional Keynesian world, the LM equation is a positive relationship between nominal interest rates and sales, given money supply. However, the curve is nearly flat on the left and nearly vertical on the right. In particular it is horizontal on the left because there is a minimum below which the nominal interest is unlikely to go. For low income, then movements in the IS curve do increase employment without affecting the rate of interest. On the other hand, a monetary expansion cannot further reduce the nominal interest rate. The economy is in a trap: whatever liquidity the authorities inject is absorbed by agents without affecting the nominal interest rate and hence income.

In a New Keynesian model, the nominal interest rate is the policy instrument and there is no explicit reference to the liquidity of the economy. Still, the interest rate has a lower bound, because otherwise agents could borrow to run infinite consumption. A liquidity trap can be defined as the situation in which the nominal interest rate is zero, so monetary policy loses its standard tool.

The zero lower bound has a direct implication for our AS-AD graphical analysis. Lowering the nominal interest rate shifts the AD curve upwards but only to the zero lower bound limit. As Figure 11 shows there is an \( AD_0 \) equation that corresponds to the lower bound and constrains any movement of AD upward.

As in the Keynesian model, the liquidity trap is a state of depression. But the intertemporal side of the New Keynesian model offers an alternative interpretation of the origin of a slump *cum* liquidity trap. As noted by Krugman (1998), in a liquidity trap the equilibrium real rate is negative because of poor long-run growth prospects. Pessimistic views of future growth shifts \( AD \) downward, as shown in Figure 12. This pessimism might be deep enough to leave very little scope for bringing the economy back up to the initial equilibrium, since \( AD_0 \) equation becomes binding. The best that can happen is to reach \( AD_0 \) by lowering the nominal interest rate to zero. But

---

7 When \( \eta = 0 \), \( AS \) and \( AD \) move from equilibrium \( E \) to \( E'' \), with no monetary policy intervention.

8 In a model with monetary frictions, the lower bound on short-term interest rates is the remuneration of money. If this is zero, then there is a zero lower bound. The analysis of this paper would be valid even if monetary frictions were assumed, with some caveats on how those frictions are introduced. In the pure cashless economy, money is remunerated at the same rate as bonds and there is no zero lower bound since the two interest rates always coincide. However, one can imagine that, at zero or at lower nominal interest rates, money could still preserve its function as store of value. In this case a zero lower bound would again become binding.
Figure 11: Zero lower bound on the nominal interest rate: $AD$ cannot move upward above $AD_0$ because of the zero lower bound on nominal interest rates.
in the equilibrium $E'$, the real interest rate is too high, household consumption too low and the economy still in a slump with output far below potential.

Which kind of monetary and fiscal policy can bring the economy back up to potential?

Although conventional monetary policy is constrained, the model still provides for one more instrument: the determination of long-run prices. The critical insight of the New-Keynesian solution to the liquidity trap, discussed in Krugman (1998), is that policymakers still have a policy tool namely acting on expectations of future policy actions.\footnote{See Eggertsson and Woodford (2003) for a solution in a fully dynamic model.}

In a liquidity trap, agents save too much because the current real interest rate is too high. By increasing expected prices and creating inflationary expectations, monetary policy can actually lower the real rate of interest and shift both $AD$ and $AD_0$ shift up and equilibrium $E''$ can be reached.

Figure 12: Liquidity trap. Starting from equilibrium $E$, by lowering the nominal interest rate monetary policy can at most reach equilibrium $E'$. But it can also lower the real interest rate by creating expectations of future inflation. In this case both $AD$ and $AD_0$ shift up and equilibrium $E''$ can be reached.
up increasing consumption and production. This channel is the stronger, the greater households’ preferences for intertemporal consumption substitution. But how can a central bank succeed in this policy or in general in a policy of moving aggregate spending, when it is denied short-term nominal interest rate maneuvers?

One possibility is “quantitative easing”, a strategy of expanding the balance sheet of the central bank and injecting liquidity into the economy until there is a reversal in prices and in particular in the relative intertemporal price of consumption. Since the general price index is the value of money in terms of other goods, there ought to be an amount of money in the system such as to prompt the right correction of the price level.

Bernanke (2002) discusses several of the instruments available to monetary policymakers in a liquidity trap. The central bank can: 1) expand the scale of its asset purchases and its menu, 2) commit to holding nominal interest rates to zero for a long period of time, 3) announce explicit ceilings for yields on long-maturity Treasury debt and enforce them by committing itself to unlimited purchases, 4) offer fixed-term loans to banks at zero or near interest, with an ample spectrum of collateral, 5) buy foreign debt. All these policies are now being tried in the United States to counteract the real economic repercussions of the subprime crisis.

The traditional IS-LM model, by contrast, implies that fiscal policy is the only effective policy instrument in a liquidity trap. In the simple New-Keynesian model presented here, let us now analyze the role of fiscal policy considering the fiscal multipliers as discussed in the previous section. We seek to determine which fiscal instrument is most effective in increasing output and at the same time narrowing the output gap vis-à-vis the natural level. As is shown in equations (22) and (23), the set of policies that increase output is $g \uparrow$, $\bar{g} \downarrow$, $\tau \downarrow$, $\bar{\tau} \downarrow$, $\tau_c \downarrow$, $\bar{\tau}_c \uparrow$, while those that narrow the output gap are $g \uparrow$, $\bar{g} \downarrow$, $\tau \uparrow$, $\bar{\tau} \downarrow$, $\tau_c \downarrow$, $\bar{\tau}_c \uparrow$. According to the two desiderata, a policy of lowering sales and payroll taxes in the short-run $\tau \downarrow$ should be avoided, because although it does increase output it worsens the output gap. All the other policies, considered separately, increase output and simultaneously diminish the gap, so they are good candidates for further investigation.

A rise in short-run public spending expands output more than a long-run reduction, while it narrows the output gap in the same proportion. However, the long-term expenditure cut has some advantages. First, an increase in $g \uparrow$ shifts the natural level of output, and AS moves downward, keeping prices down. In a depression with low inflation or deflation, keeping prices down might heighten the risk of worsening the balance sheets of firms in the presence of downward nominal wage rigidities or debt.

\[^{10}\] The analysis might also be conducted with reference to the efficient level of output, but in a depression the efficient level might be far enough away to make it a secondary objective, subordinate to bringing the economy up to potential.

\[^{11}\] In a New-Keynesian multi-period model, Eggertsson (2008b) shows that the multiplier on tax cuts goes from positive to negative once the interest rate hits zero. Instead, the multiplier on public spending becomes larger than one.
A lowering in $\tilde{g} \downarrow$ by contrast only moves AD upward; it puts no downward pressure on prices. Second, a short-run increase in public spending crowds out private consumption, whereas a cut in long-run public spending increases current private consumption. This might be desirable in a slump in which consumption contracts, given the large portion of GDP usually accounted for by consumption. Third, increasing short-run public spending is effective if it is not permanent, which might require more political effort than a policy of just trying to reduce fiscal expenditure in the long run. Finally, if there are problems of fiscal sustainability or if lump-sum transfers are not available, then a short-run increase in public spending will have to be accompanied by a rise in taxes, which undercuts its effectiveness.

Reducing short-run consumption taxes $\tau_c \downarrow$ and increasing long-run consumption taxes $\tilde{\tau}_c \uparrow$ both produce similar effects on the output gap but the former policy also moves AS downward, with the problems pointed out above, while the latter shifts only the AD. A reduction in long-run taxes, $\tilde{\tau} \downarrow$, shifts the AD curve only.

In general, what this analysis indicates is that there is a better mix of fiscal policy instruments that can stimulate demand and get out of the recession cum liquidity trap. This mix includes mainly long-run fiscal policies with expansionary effects on short-run consumption: cut in future taxes and future public expenditure and an increase in future consumption taxes work in the same direction.\footnote{Similarly, Eggertsson (2008b), in a more sophisticated model, argues for policies that shift the AD equation. In a multi-period model with a persistent state of liquidity trap and sticky prices, Eggertsson (2008a) finds that even policies that increase the mark-up can expand output since they create expectations of inflation and lower the real rate of interest, stimulating demand.} Moreover, such policies are sustainable as they comply with the government budget constraint. By contrast, an increase in current public expenditure or a decrease in current taxes should be accompanied by higher taxes or lower public expenditure in the future. But higher taxes in the future are among the policies that most sharply reduce current output and the output gap. Moreover, if it is to be effective on the output gap, expansionary short-run fiscal policy has to be reversed in the long run.

10 Optimal monetary policy

Having seen how to escape from the liquidity trap, we can now return to normal conditions, to characterize the way in which monetary policy should react to disturbances. Our New Keynesian model provides an obvious solution as microfoundations offer a natural welfare criterion, based on consumers’ utility that serves to evaluate alternative allocations following from different policy rules. Interestingly, the resulting objective can be written similarly to those that have been assumed on an ad-hoc basis by the literature on monetary policy in the 1980s.

Woodford (2003, ch. 3) shows that a second-order approximation of (1) implies a
loss function of the form
\[
L = \frac{1}{2}(y - y_e)^2 + \frac{1}{2\kappa}(p - p^e)^2. \tag{24}
\]

To serve consumers’ interests, monetary policymakers should choose their instrument to minimize deviations of output from the efficient level and fluctuations of prices from the expected level. The relative weights to assign to the two objectives depends significantly on the fraction of firms that adjust.\(^{13}\) The smaller the fraction, the greater the weight to give to price stability. The greater the elasticity \(\theta\), the greater the weight to assign to the price stability objective. A similar relationship holds for the intertemporal elasticity of substitution \(\sigma\). According to Svensson (2007a), (24) represents a flexible inflation-targeting objective with price and output targets.\(^{14}\)

But, how can an inflation-targeting central bank achieve a minimum target for the loss function? One way could be to specify a rule for the instrument (the interest rate) as a function of observable variables. An appropriate “Taylor rule” of this sort for our model would be
\[
i = \bar{i} + \psi_y(y - y_e) + \psi_\pi(p - p^e)
\]

for non-negative parameters \(\bar{i}, \psi_y\) and \(\psi_\pi\).\(^{15}\) The monetary policymakers could then choose the parameters \(\psi_y\) and \(\psi_\pi\) so such that the equilibrium outcome given by the interaction of the interest rate rule together with the AS-AD model will minimize the loss function. But it would be a fortuitous if this policy were optimal for all circumstances. Indeed, the optimal parameters \(\psi_y\) and \(\psi_\pi\) are likely to depend on the properties of the shocks. Yet there is a more direct way to characterize optimal policy, which delivers a simple operational targeting rule to follow under any and all circumstances, discussed in detail in Giannoni and Woodford (2002).

Optimal policy can be simply characterized by substituting (17) into (24) to obtain
\[
L_t = \frac{1}{2}(y - y_e)^2 + \frac{\theta}{2\kappa}(y - y_n)^2,
\]

which can be easily minimized with respect to \(y\) to obtain
\[
(y - y_e) + \theta\kappa(y - y_n) = 0,
\]

and re-written as
\[
(y - y_e) + \theta(p - p^e) = 0. \tag{25}
\]

\(^{13}\)The approximation holds around an efficient steady state in which \(\mu = 0\). In the more general case, treated in Benigno and Woodford (2005), the welfare-relevant output does not necessarily coincide with \(y_e\) and obviously the conclusions of this section are going to change. This affects mostly the analysis of the optimal response with respect to mark-up shocks. Benigno and Woodford (2003) study the more general case in which lump-sum taxes are not available.

\(^{14}\)For a discussion of inflation targeting, see also Bernanke and Mishkin (1997).

\(^{15}\)See Taylor (1993).
This is a linear relationship between the output gap and the price-stability objective. Svensson (2007a) calls (25) an optimal targeting rule that describes the “equality of the marginal rates of transformation and substitution between the target variables in an operational way”. This rule is robust to all types of shock and does not depend on their properties. To put it simply, the policy instrument, i.e. the nominal interest rate, should be moved in a way to achieve equation (25) in any case, whether the shock is hit to mark-up, productivity or public spending and whether it is temporary, permanent, or expected.

The monetary policymaker is willing to tolerate price inflation insofar as it coincides with a contraction in output, and vice versa. It is a nice coincidence that this targeting rule involves a negative relationship between prices and output that can be plotted in the same graph as AS and AD. Let us call (25) the IT equation. The slope is given by $-\frac{1}{\theta}$, and IT crosses the point $(p^e, y_e)$. In particular IT is flatter than AD whenever $\theta > \sigma$. And this is the empirically relevant case.\textsuperscript{16}

While it is natural in a microfounded model to assume consumers’ utility as the gauge of welfare, in practice this assumption is not self-evident or automatic. Society may well give the central bank different set of preferences and objectives. A central bank might have a general mandate for price stability, but no explicit guidance on the relative weights of output stabilization and price stability. In fact, at the level of the monetary policy committee there may be different preferences that shape the final policy decision. To account for this, we can define a general flexible inflation targeting policy of the form

$$L = \frac{1}{2}(y - y_e)^2 + \frac{1}{2\kappa}(p - p^e)^2,$$

(26)

for a generic non-negative parameter $\phi$ that measures the preferences of the policymaker on the inflation-output trade-off. In this case the optimal targeting rule requires

$$(y - y_e) + \phi(p - p^e) = 0$$

where the parameter $\phi$ governs the slope of IT. A central bank with great concern for price stability will have a flatter IT curve, while one mainly worried about output stability would have a steeper curve.

Coming back to mark-up shocks (Section 7), let us focus on a central bank concerned for price stability, as it should be under the metric given by the welfare of the consumer, with a relatively flat IT. Following a temporary increase in the mark-up, without monetary policy intervention the economy reaches equilibrium $E'$ in Figure 13.

Optimal monetary policy should place the economy on the IT curve, which in this case does not move since $y_e$ did not move, at the intersection with AS. It should

\textsuperscript{16}Indeed, $\theta$ is related to the mark-up: a value around 8 gives a 15% mark-up; $\sigma$ is the elasticity of substitution in consumption which is usually assumed close to 1.
Figure 13: A temporary mark-up shock: $\mu_\theta \uparrow$. \( AS \) shifts to \( AS' \). The equilibrium moves from \( E \) to \( E' \). \( IT \) equation describes the optimal combination of prices and output according to the preferences of the monetary policymakers. The optimal equilibrium is \( E'' \) where \( IT \) intersects \( AS' \). Monetary policy should raise the nominal interest rate to move the equilibrium from \( E' \) to \( E'' \).
reach \( E^* \) by raising the nominal interest rate. A central bank less concerned for price stability, with \( IT \) steeper than \( AD \), should lower the nominal interest rate to achieve its own optimal allocation.

The optimal policy analysis of this section can be repeated for productivity shocks, in which \( IT \) shifts to cross the optimal equilibrium point of stable prices and contemporaneous closure of the output gap, as discussed, and for public-spending shocks as well. All the other cases are left to the reader.
References


