1. Sticky Price Models: The Policy Implication

Consider the models with sticky prices, such that some firms don’t make immediate responses to the changes in the price level (Surely Calvo-Yun model is one of them).

a) Explain the difference between ex ante and ex post mark-up.

Ex ante mark-up is the mark-up the firm desires at the time when it sets its price, which could be expressed as

\[ E_t [P_s(z)] = E_t [(1 + \mu_s)P_sMC_s] \]

in which \( s > t \). Ex post mark-up is the mark-up when the uncertainties are resolved, which could be expressed as

\[ P_t(z) = (1 + \mu'_s)P_sMC_s \]

in which \( s > t \), and \( P_t(z) \) is the firm’s price at time \( s \) which was already fixed at time \( t \). Suppose that there is an unexpected increase in aggregate demand, and therefore an increase in the demand of labor which drives up \( MC_s \). Since the firm’s price was already fixed, then its ex post mark-up goes down. So with sticky prices in assumption, we have the counter-cyclical mark-ups.

b) Explain how monetary policy affects the real economy.

Now money matters because of sticky price settings. For any change in price level, due to monetary policy, the firms’ ex post mark-ups differ from each other. Therefore,
the firms differ from each other on their relative prices, and this leads to a dispersion in the consumption of differentiated goods.

2. Calvo-Yun Model: The Driving Forces

Consider the Calvo-Yun model with staggered price setting.

a) Show that increases in output have a positive impact on inflation.

The aggregate supply relation in Calvo-Yun model is captured in the new Keynesian Phillips curve, which could be expressed as following

\[ \pi_t = \kappa m c_t + \beta E_t \pi_{t+1}. \] (1)

The increase in output reflects the excess demand, which in turn, causes scarcity of output and running capital at high intensity. This results in higher marginal cost, hence higher prices and greater inflation.

b) Explain, why the resulting aggregate supply curve is forward looking.

Because the price is sticky such that the firms set prices in advance. Since the firms have market power, so if the price is flexible the price that maximizes one firm’s profit is a constant mark-up over marginal cost. But when the price is sticky, the firm’s mark-up would be affected when there are any changes in the price level, during the periods when the firm is not able to adjust its price. Therefore, whenever the firm has the chance to set its price, it has to take into account both current real marginal cost and expected future real marginal cost — so that the resulting aggregate supply curve is forward looking.

c) Explain how the economy is distorted by monopolistic competition and staggered price setting. Provide some intuitions on how economic policies may restore the efficiency of equilibrium allocations.

The Distortion Related to Monopolistic Competition

Monopolistic competition distorts factor prices, hence the agent’s intratemporal
decisions on consumption and labor,

\[
\frac{\partial u_t}{\partial N_t} = \frac{W_t}{P_t} = \frac{1}{1 + \mu} \frac{\partial Y(z)}{\partial N(z)}.
\]

This suggests that it would be optimal to subsidize the employment cost. Suppose that at the rate \( \tau \) the employment is subsidized, then

\[
-\frac{\partial u_t}{\partial C_t} = (1 + \tau) \frac{W_t}{P_t} = 1 + \tau \frac{\partial Y(z)}{1 + \mu} \frac{\partial N(z)}{\partial C_t}
\]

and the optimality would be restored if the social planner sets \( \tau = \mu; \)

**The Distortion Related to Staggered Price Setting**

It is known that the output level of a monopolistically competitive firm, \( Y_t(z) \), is determined by the aggregate output \( Y_t \) and the relative price \( \frac{P_t(z)}{P_t} \) in a way that

\[
Y_t(z) = \left[ \frac{P_t(z)}{P_t} \right]^{-\epsilon} Y_t.
\]

Apply this expression to calculate the aggregate output \( Y_t^\epsilon \)

\[
Y_t^\epsilon = \int_0^1 Y_t(z) dz = \int_0^1 \left[ \frac{P_t(z)}{P_t} \right]^{-\epsilon} Y_t dz
\]

\[
= Y_t \int_0^1 \left[ \frac{P_t(z)}{P_t} \right]^{-\epsilon} dz,
\]

\[
A_t N_t^a K_{t-1}^{1-a} = Y_t \int_0^1 \left[ \frac{P_t(z)}{P_t} \right]^{-\epsilon} dz,
\]

and the last step uses the fact that the production function is constant return to scale. Now let’s define a new variable to finish the aggregation of production

\[
s_t = \int_0^1 \left[ \frac{P_t(z)}{P_t} \right]^{-\epsilon} dz,
\]

which measures the gap between aggregate output of intermediate goods and final
goods, i.e.

\[ Y_t^z = A_t N_t^z K_{t-1}^{1-\alpha} = s_t Y_t. \]

Obviously \( s_t = 1 \) if there is no price dispersion, which is caused by fluctuations in price level. Define \( \zeta_t = \left[ \frac{P_t(z)}{P_t} \right]^{1-\epsilon} \), and obviously \( s_t = \int_0^1 \zeta_t^{\frac{\epsilon}{\epsilon-1}} dz \). Notice that

\[
\left( \int_0^1 \zeta_t dz \right)^{\frac{\epsilon}{\epsilon-1}} = \left\{ \int_0^1 \left[ \frac{P_t(z)}{P_t} \right]^{1-\epsilon} dz \right\}^{\frac{\epsilon}{\epsilon-1}} = P_t^{\epsilon} \left\{ \int_0^1 P_t(z)^{1-\epsilon} dz \right\}^{\frac{\epsilon}{\epsilon-1}} = 1
\]

using the definition of price index for the last step, one can see that

\[
1 = \left( \int_0^1 \zeta_t dz \right)^{\frac{\epsilon}{\epsilon-1}} \leq \int_0^1 \zeta_t^{\frac{\epsilon}{\epsilon-1}} dz = s_t
\]

by Jensen’s inequality because \( \epsilon > 1 \) and \( \frac{\epsilon}{\epsilon-1} > 1 \), and the equality holds only for \( \zeta_t \) being constant, i.e. \( P_t(z) = P_t, \forall z \in [0,1] \) — when there exists no price dispersion.

Since \( s_t \) is bounded below by 1, the output level, i.e. the production of the final goods, is distorted by the factor of \( s_t \) due to the existence of price dispersion. Since the prices are adjusted in a staggering manner in our economy, the only way to wipe out such inefficient price dispersion is to keep the price level constant, such that the firms don’t have to adjust their prices at all. Therefore, it would a desired policy to stabilize the price level, i.e. to eliminate inflation, in this economy.

**d) Show that stabilization of output and inflation are no conflicting goals.**

There is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero. Rewrite (1) in terms of output gap

\[
\pi_t = \kappa (\gamma + \gamma_n) x_t + \beta E_t \pi_{t+1}, \quad (2)
\]

then when output is stabilized such that \( x_{t+i} = 0 \) for all \( i > 0 \), then \( \pi_{t+i} = 0 \), and the same argument holds when inflation is maintained at zero. Because firms adjust prices in a
staggered manner, inflation generates a costly dispersion of prices; the central bank can eliminate this source of distortion by ensuring price stability. When firms do not need to adjust their prices, the fact that prices are sticky is no longer relevant, and the output is thus stabilized.

3. New Keynesian Perspective

Consider the “new Keynesian perspective” featured by an IS curve

\[ x_t = -\phi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t, \]

where \( x_t \) is the output gap, and a Phillips curve

\[ \pi_t = \lambda x_t + \beta E_t\pi_{t+1} + u_t, \]

\( g_t \) and \( u_t \) are shocks that obey \( g_t = \mu g_{t-1} + \hat{g}_t \) and \( u_t = \rho u_{t-1} + \hat{u}_t \), where \( \mu \geq 0, \rho \leq 1 \) and \( \hat{g}_t \) and \( \hat{u}_t \) are i.i.d. random variables with zero mean and constant variances.

The policy objective is given by

\[
\min E_t \left[ \sum_{i=0}^{+\infty} \beta^i \left( \alpha x_{t+i}^2 + \pi_{t+i}^2 \right) \right].
\]

a) Explain the “new” IS curve and the forward looking Phillips curve.

These equations constitute the non-policy block of the basic new Keynesian model. That block has a simple recursive structure: the new Keynesian Phillips curve determines inflation given a path for the output gap, whereas the IS curve determines the output gap given a path for the actual real rate. Solve \( x_t \) and \( \pi_t \) forward and get

\[
x_t = -\phi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t
= E_t \left[ \sum_{i=0}^{+\infty} -\phi(i_{t+i} - \pi_{t+i+1}) + g_{t+i} \right],
\]

— Note that \( r_t = i_t - E_t\pi_{t+1} \) is the expected real return on a one period bond (i.e. the real interest rate), it emphasizes the fact that the output gap is proportional to the sum
of current and anticipated real interest rate;

\[
\pi_t = \lambda x_t + \beta E_{t+1} \pi_{t+1} + u_t \\
= E_t \left[ \sum_{i=0}^{+\infty} \beta^i (\lambda x_{t+i} + u_{t+i}) \right],
\]

— Inflation depends on current and expected output gap, hence marginal cost, as explained in Problem 2 b).

b) Explain the policy objective. What are the differences from a Barro-Gordon type policy objective?

The policy objective is to minimize the fluctuations in output and inflation, and \( \alpha \) determines the importance of output fluctuations relative to inflation fluctuations.

The policy objective looks a lot like the standard quadratic loss function used in Barro-Gordon model. There are, however, two critical differences. First, the output gap is measured relative to equilibrium output under flexible prices. In the traditional Barro-Gordon model, the output variable was more commonly interpreted as output relative to trend or output relative to the natural rate of output. The natural rate of output varies with productivity shocks, but it is not the same as the flexible-price equilibrium level of output used to define the gap variable \( x_t \). A second difference is the reason inflation variability enters the loss function. When prices are sticky, inflation results in an inefficient dispersion of output among the individual producers. The representative households utility depends on its consumption of a composite good; faced with a dispersion of prices for the differentiated goods produced in the economy, the household buys more of the relatively cheaper goods and less of the relatively more expensive goods. Because of diminishing marginal utility, the increase in utility derived from consuming more of some goods is less than the loss in utility due to consuming less of the more expensive goods. Hence, price dispersion reduces utility. Similarly, dispersion on the production side is costly. The increased cost of producing more of some goods is greater than the cost saving from reducing production of other goods. For these reasons, price dispersion reduces utility, and, when each firm does not adjust its price every period, price dispersion is caused by inflation.

However, in Barro-Gordon model, the efficiency distortion that leads to \( x^* \) was used to motivate the presence of an overly ambitious output target in the central banks objective function. As a consequence, the presence of \( x^* > \bar{x} \) implies that a central bank acting under discretion to maximize the policy objective would produce an average
inflation bias, which is the source of the Barro-Gordon inflation.

c) Derive the optimal discretionary policy for rational expectations and show that there is a short-run trade-off between inflation and output variability.

The optimal discretionary policy under rational expectations is derived in two stages:

1. Choose $x_t$ and $\pi_t$ in order to maximize the policy objective, given the new Keynesian Phillips curve;

2. Given these optimal values of inflation and output gap, determine the optimal setting of the interest rate implied by the IS curve, i.e. the interest rate that will support the inflation and output levels determined in the last step.

With discretionary policy the central bank cannot manipulate the beliefs, therefore it takes the expectation of the private sector as given in solving the optimization problem (Surely, conditional on the central bank’s optimal rule, the private sector forms the beliefs rationally). As a consequence, policy decisions today do not affect future expectations, which implies that the maximization problem can be written as

$$\min E_t \left[ \sum_{i=0}^{+\infty} \beta^i (\alpha x_{t+i}^2 + \pi_{t+i}^2) \right] = \alpha x_t^2 + \pi_t^2 + F_t,$$

s.t.

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t = \lambda x_t + f_t,$$

in which

$$F_t = E_t \left[ \sum_{i=1}^{+\infty} \beta^i (\alpha x_{t+i}^2 + \pi_{t+i}^2) \right],$$

$$f_t = \beta E_t \pi_{t+1} + u_t$$

are the terms taken as given in the optimization, capturing the ideas such that

- Future inflation and output are not affected by today’s actions;
- The central bank cannot directly manipulate expectations.

By the first order conditions one can see that

$$x_t = -\frac{\lambda}{\alpha} \pi_t.$$

(3)
Remember that the inflation target here is zero, so if inflation is positive, the central bank needs to create a recession to reduce inflation, i.e. “leaning against the wind” argument. To achieve this, whenever inflation is above target the central bank contracts demand by raising the interest rate. How aggressive such action is depends on

- The gain in reduced inflation per unit of output loss (positive effect);
- The weight on output fluctuation in preferences (negative effect)

— The short-run trade-off between inflation and output variability.

**d) Using discretionary policy: How must the interest rate respond to a rise in expected inflation?**

Now solve the nominal interest rate rule by applying the first order condition in the new Keynesian model. Insert (3) into new Keynesian Phillips curve

\[
\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t \\
= \lambda \left( -\frac{\lambda}{\alpha} \pi_t \right) + \beta E_t \pi_{t+1} + u_t, \\
\pi_t = \frac{\alpha \beta}{\alpha + \lambda^2} E_t \pi_{t+1} + \frac{\alpha}{\alpha + \lambda^2} u_t.
\]

Then in order to pin down \( E_t \pi_{t+1} \), we impose the rational expectation condition of the private sector, i.e. the expectations should be consistent with the predictions of the model and the updated version of the equation above should still hold

\[
\pi_{t+1} = \frac{\alpha \beta}{\alpha + \lambda^2} E_{t+1} \pi_{t+1} + \frac{\alpha}{\alpha + \lambda^2} u_{t+1}.
\]

Take expectations on both sides and get

\[
E_t \pi_{t+1} = \frac{\alpha \beta}{\alpha + \lambda^2} E_t \left[ E_{t+1} \pi_{t+2} \right] + \frac{\alpha}{\alpha + \lambda^2} E_t u_{t+1}.
\]

Recall that the supply shock follows \( u_t = \rho u_{t-1} + \hat{u}_t \), then

\[
E_t u_{t+1} = \rho u_t + E_t \hat{u}_{t+1} = \rho u_t.
\]

Therefore \( E_t \pi_{t+1} \) can be computed

\[
E_t \pi_{t+1} = \frac{\alpha \beta}{\alpha + \lambda^2} E_t \left[ \pi_{t+2} \right] + \frac{\alpha \rho}{\alpha + \lambda^2} u_t.
\]
and substitute $E_t \pi_{t+1}$ in the expression for $\pi_t$ to get

\[
\pi_t = \frac{\alpha \beta}{\alpha + \lambda^2} \left( \frac{\alpha \beta}{\alpha + \lambda^2} E_t [\pi_{t+2}] + \frac{\alpha \rho}{\alpha + \lambda^2} u_t \right) + \frac{\alpha}{\alpha + \lambda^2} u_t
\]

\[
= \left( \frac{\alpha \beta}{\alpha + \lambda^2} \right)^2 E_t [\pi_{t+2}] + \left( \frac{\alpha}{\alpha + \lambda^2} \right)^2 \beta \rho u_t + \frac{\alpha}{\alpha + \lambda^2} u_t
\]

\[
= \frac{\alpha}{\alpha + \lambda^2} \frac{1}{1 - \frac{\alpha \rho \beta}{\alpha + \lambda^2} u_t},
\]

in which the last step is ensured by the transversality condition. Update $\pi_t$ one period forward to get

\[
E_t \pi_{t+1} = E_t \left[ \frac{\alpha}{\alpha + \lambda^2} \frac{1}{1 - \frac{\alpha \rho \beta}{\alpha + \lambda^2} u_t} \right]
\]

\[
= \frac{\alpha}{\alpha + \lambda^2} \frac{\rho}{1 - \frac{\alpha \rho \beta}{\alpha + \lambda^2} u_t}.
\]

By equation (3) one can see that

\[
x_t = -\frac{\lambda}{\alpha} \pi_t
\]

\[
= -\frac{\lambda}{\alpha + \lambda^2} \frac{1}{1 - \frac{\alpha \rho \beta}{\alpha + \lambda^2}} u_t
\]

\[
= -\frac{\lambda}{\alpha + \lambda^2} \frac{1}{1 - \frac{\alpha \rho \beta}{\alpha + \lambda^2}} u_t
\]

Update $x_t$ one period forward to get

\[
E_t x_{t+1} = -\frac{\lambda}{\alpha + \lambda^2} \frac{\rho}{1 - \frac{\alpha \rho \beta}{\alpha + \lambda^2}} u_t.
\]

Then insert everything into the IS curve

\[
x_t = -\phi \left( i_t - E_t \pi_{t+1} \right) + E_t x_{t+1} + g_t
\]

\[
i_t = E_t \pi_{t+1} - \frac{1}{\phi} x_t + \frac{1}{\phi} E_t x_{t+1} + \frac{1}{\phi} g_t
\]

\[
= E_t \pi_{t+1} + \frac{1}{\phi} \frac{\lambda}{\alpha \rho} E_t \pi_{t+1} - \frac{1}{\phi} \frac{\lambda}{\alpha} E_t \pi_{t+1} + \frac{1}{\phi} g_t
\]
showing how the interest rate responds to a rise in expected inflation. The result is that the central bank creates a recession to combat the inflation, as shown in Figure 1.

\[ E_t \pi_{t+1} \left[ 1 + \frac{\lambda(1 - \rho)}{\alpha \phi \rho} \right] + \frac{1}{\phi^t} \]

Figure 1: Central Bank’s Response to a Rise in Expected Inflation